Information Economics

Course Manual 1036M / EBC 4025
School of Business and Economics
Maastricht University

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1 Introduction

This course deals with the Economics of Information, i.e. with the question of how people decide whenever their information is (in)complete, how they acquire new information, how they learn, and how relationships develop if the different parties have different information about their counterparts and/or the environment. Stated differently, we investigate how an economy adapts to new information, and how this information is disseminated, absorbed and used throughout the economy.

Them "Bank of Sweden Award in Economic Sciences in Memory of Alfred Nobel" has been awarded to George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz in 2001 for their contributions to the Economics of Information. The fathers of Information Economics are mainly responsible for three contributions that will also serve as a guideline for this course.[1]

- Information Economics questions standard paradigms of economics. Information is not just a commodity like many others. If information is distributed unequally over an economy (or a relationship) a lot of paradigms do no longer hold. Market equilibria may not exist, and if they exist they are not necessarily Pareto optimal. Moreover the distribution of income matters, and markets are not necessarily clearing.

- Information Economics explains empirical puzzles of standard economics. Taking asymmetric distributions of information as a starting point, many empirical puzzles can be explained. For instance, unobservable management efforts can explain wage differentials that do not reflect different productivity levels (the efficiency wage discussion). The financial structure of a firm can signal the quality of a project (the capital structure puzzle). Or auction may lead to efficient resource allocations while bilateral bargaining does not (a violation of Coase Theorem). In general: Information Economics helps to explain why market (or institutional) designs matter.

- Information Economics provides an applicable toolkit to analyze relevant economic settings. This course will discuss several applications of Information Economics beyond their main scope. We will use the toolbox to discuss recent developments in the design of health insurance contracts, electronic marketplaces, or agreements for technology transfers.

2 Goals

In this course we study the role of information in three dimensions.

[1] A comprehensive review of the impact of Information Economics on economic thinking can be found in Stiglitz (2000).

- *How is information transmitted in (economic) relations?*

  We investigate different (contractual) relationships with asymmetrically distributed information and analyze the respective (private) benefits and (social) losses. Moreover, we discuss how information is (or can or should be) transmitted in these relationships.

- *How is information processed?*

  If we know how economic relations (or markets) transmit information, an obviously related question is how this information is processed by the respective receiver. This leads to theories of learning and up-dating (and their empirical qualifications from cognitive psychology). Finally, we will discuss how the strategic (abuse) of information influences processing (and transmission).

- *How does market design influence information transmission?*

  If different economic relations (or different institutional arrangements) influence information transmission in the market, it is important to analyze various market designs and their efficiency properties.

The course aims at the provision of a working knowledge in all three dimensions that enables students to identify and analyze problems of information transmission in economic relationships, to evaluate their welfare consequences, and to recommend institutional improvements.

3 Structure

The course contains bi-weekly meetings of two hours each making a total of 14 educational meetings. The first meeting will be spend on an introduction, an overview of basic concepts, and commitments. We continue with 9 Round Table sessions that provide an introduction and a discussion of the relevant theoretical tools. Subsequently, participants will conduct research for their final assignment (for one week instead of the respective meetings, there will be the opportunity to discuss the research with the tutor). In the final session, students will present a draft version of their final assignment. The Schedule is detailed in Appendix A.

3.1 Round Table Sessions

In every Round Table Session a chairperson (allocation of this job will be conducted in the first meeting) organizes a discussion of the assigned reader. Experience shows that it facilitates matters to post questions in advance on the discussion board (of the respective group) such that the chair can organize the different problems. Needless to say that the understanding of the concepts discussed in the reader is absolutely essential in order to understand the literature which is relevant for the final assignment.

3.2 Problem Sets

In each Round Table Session we discuss a Problem Set that provides a useful benchmark for your understanding of the reader. Prepare solutions to the Problem Set that are blackboard-ready. The Problem Sets can be found in Appendix B. Problem Sets marked with * have to be submitted to the tutor before the respective meeting and will be subject to grading.

3.3 Final Assignment

For the final assignment every student has to select a topic that deals with the Economics of Information. A list of potential research questions is enclosed (see Appendix C and - for further inspiration - a list of additional articles in Appendix D) - independent ideas, however, are more than welcome. Experience shows that it pays off to think about the final assignment throughout the course - to discuss the topic with your fellow students and/or the tutor, to conduct a literature research and to organize the paper. Do not postpone everything to the end of the course! You loose plenty of opportunities for discussions, and also the opportunity to let the problem mature. The course schedule leaves space for continuous work on the final assignment - use it! Research Questions should be claimed on the discussion board (in ELEUM). We apply the usual first-come-first-serve rule. Every topic can only be investigated in one paper. The evaluation criteria for the final assignment are the following formal and content related points:[2]

- The paper can be joined work with one other student. Grading will not discriminate between the contributions of the different authors. If the paper is single authored it must not exceed 17 pages (without front-matter and references), if it is joined it must not exceed 30 pages - double spaced, script 12. Papers not fulfilling these criteria will not pass.

- In week 6 (after Round Table 9), you can discuss questions regarding your final assignment with the tutor. In one of these sessions (or - preferably - via email), a table of content for the final assignment - and a plan for the presentation - should be approved by the tutor.

- The paper should deal with a clearly identified research question that can be answered with tools learned in this course.[3] Typical research questions can be found in the Appendix, new

[2] However, they may not apply in exactly the same way to all feasible topics.
[3] Successful examples will be published on ELEUM.

(good) research questions are clearly an achievement as such, but should be discussed in advance with the tutor.

- The paper should identify the informational problems of the respective research question and should relate it to the models discussed in the course.
- Discuss the feasibility of empirical test in the respective environment. How would these tests look like? What data-sets would be needed?
- Is there an obvious connection of the research question to experimental or behavioral economics. Will cognitive limits of information processing or non-standard preference structures qualify your results?
- After you assessed the robustness (and the opportunity to check empirical validity) of your findings, you should venture real-life (policy) recommendations.

The final assignment has to be in the mail-box of your tutor until Friday, March 26^{th}**, 2010, at noon.**

3.4 Paper Presentations

Every (group of) student(s) has to present a draft version of their final assignment. A presentation should not last longer than 10 minutes (per person) plus a discussion. The presentation is subject to grading. The presentation introduces the respective research question, reviews the relevant literature (very briefly), and tries to give a preliminary view on the research for the final assignment. The main goal of this presentation is twofold. First, your fellow students should get an idea about this particular area of information economics (that has not been touched substantially throughout the course). Hence, presenters should make an attempt to introduce the research question and the relevant model(s) starting from insights that are familiar to every participant. Second, the presentation should open up a discussion about the respective research project - with fellow students and the tutor. Presenters should therefore try to stimulate feedback and critical comments.

4 Grading Policies

Your evaluation will be based on four parts: your participation in the group (30%), the submitted problem sets (30%), your presentation (10%), and the final assignment (30%). An insufficient mark for any of these parts cannot be compensated. As a result, the passing requirement is 55% for each part separately.

The evaluation of your participation will be based on: your presence, both physically and mentally, which is expected in every session; your contributions in raising meaningful questions; your contributions to bringing out problems; your contributions to solving problems of interpretation (of yourself and others) and the Problem Sets; and your performance as chairperson. A trend for the participation grade will be published after the first half of the course. Presentations will be graded directly after the respective session. The mark for the final assignment and the overall grade are published on ELEUM.

You are expected to be present in every meeting. If you cannot attend a certain meeting for a serious reason, please contact the block coordinator. If you miss a Round Table Session you have to hand-in a fully worked-out solution of the entire Problem Set that was scheduled for the respective meeting before the next session.

5 Literature List

The core reference for this course is the comprehensive:

- Macho-Stadler, I. and D. Perez-Castrillo, An Introduction to the Economics of Information, 2nd edition, Oxford University Press 2001.

In addition to Macho-Stadler and Perez-Castrillo (2001), we will provide all papers mentioned in the reference list below as downloads in the section Course Material (Readings) of the ELEUM site of this course.

For assumed pre-existing knowledge on microeconomics, we refer to your textbook(s) on microeconomics. In Appendix E and F, you will find a brief refresher in expected utility theory and the theory of risk aversion. We will assume knowledge of these concepts throughout the course. In particular, all course participants are asked to check their mathematical skills on the exercise in Appendix G. The mathematics of constrained optimization is a necessary pre-requisite for the entire course. A detailed solution to the exercise with exhaustive explanations can be found in Appendix A1.2 in Kreps, D., A Course in Microeconomic Theory, Princeton University Press, 1990.

6 The Planning Group

The planning group consists of

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- Rene Saran (e-mail: r.saran@maastrichtuniversity.nl, phone: 83763, office: A 0. 07 (TS53)).

The course will be coordinated and tutorial meetings are conducted by Çağatay Kayı.

7 References

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A Schedule

1. Block Opening: Refresher in Choice under Uncertainty

   - Overview and Technicalities
   - Throughout this session we will review the essence of the theory of choice under uncertainty which is the basic toolkit for Information Economics. A preparatory look at the respective reader will do no harm. For students with a bachelor in Econ the material should be familiar, for students with a different background we recommend a closer look. In particular, students are asked to solve the exercise in Appendix G. It will be impossible to follow this course without mastery of the calculus of constrained optimization.
   - Reader: Appendix E, F and G.

2. Round Table 1: Incentives, Contracts and Risk-Sharing

   - First, we focus on an example how workers respond to incentives (Lazear, 2000). Then, as a benchmark, we deal with the simplest contractual relationship - an agreement on risk-sharing under complete information. We will analyze optimal risk-sharing and apply our findings to some real-life settings.
   - Reader: Lazear, E. (2000), Macho-Stadler and Perez-Castrillo (2001) (henceforth, MSPC) ch. 1. and ch. 2.
   - Problem Set 1.

3. Round Table 2: Moral Hazard

   - This session introduces the standard model for contracts with hidden actions (moral hazard). We analyze optimal contract structures and welfare implications if actions are not observable and apply our findings to performance dependent wage agreements.
   - Reader: MSPC ch. 3.1-3.3.
   - Problem Set 2*(to be submitted for grading).

4. Round Table 3: Adverse Selection

   - This session introduces the standard model for contracts with hidden characteristics (adverse selection). We analyze optimal contracts and welfare implications if characteristics are not observable and apply our findings to monopolistic price discrimination.
   - Reader: MSPC ch. 4.1 and 4.2.

- Problem Set 3.[4]

5. Round Table 4: Information and Insurance Markets

    - This session provides a detailed discussion of one of the first papers in Information Economics: The insurance market model by Rothschild and Stiglitz (1976). We will use this model to identify conditions for market failures due to asymmetric information.
    - Reader: Rothschild and Stiglitz (1976).
    - Problem Set 4.

6. Round Table 5: Empirical Tests of Contract Theory

    - In this session, we deal with the recent literature on empirical tests of contract theory. Hereby, we will focus on the detection and relevance of asymmetric information effects in insurance, labor, and technology transfer markets.
    - Reader: Chiappori and Salanie (2002) p.1-43 (except sections 3.2 - 3.4).
    - Problem Set 5.

7. Round Table 6: Learning and Information Processing

    - We continue with an introduction to the theory of rational learning behavior (i.e Bayesian updating) and apply the knowledge to an analysis of probabilistic test designs. Furthermore, we discuss some classics in cognitive psychology that illustrate the limits of rational learning behavior.
    - Reader: van Zandt ch. 2.1 and 2.2, Kahnemann and Tversky (1971).
    - Problem Set 6.

8. Round Table 7: Strategic Information Transmission (Signalling)

    - Endowed with a theory about learning, we are ready to address the strategic (ab)use of information in signalling games. As a key example, we consider Spence job market model with education as a signal.
    - Reader: MSPC ch. 5.1-5.4.
    - Problem Set 7* (to be submitted for grading).

9. Round Table 8: Fairness and Contract Design

[4] Obviously, there will be no sessions scheduled for Carnival week.

- In this session we discuss additional insights into incentive and signaling theory provided by laboratory experiments. In a recent paper Fehr, Klein, and Schmidt (2007) show how principals design contracts in laboratory experiments and thereby anticipate social preference structures.
- Reader: Fehr, Klein, and Schmidt (2007).
- Problem Set $8^{\star}$ (to be submitted for grading).

10. Round Table 9: Auctions

    - We conclude the round table sessions with an analysis of auctions, one of the most popular trade institutions in our days - and one of the most ideal playgrounds for economists.
    - Reader: Milgrom (1989).
    - Problem Set 9.

11. Paper Consultation Session 1 (Tuesday, March 16^{th}, 2010).

12. Paper Consultation Session 2 (Friday, March 19^{th}, 2010).[5]

13. Presentation Session (Tuesday, March 23^{rd}, 2010).[6]

[5] You do not have to participate Paper Consultation Sessions. If you want to, you should get an appointment from your tutor to have a consultation on your final assignment.

[6] Presentation Schedule will be published on Eleum in due course.

B Problem Sets

Problem Set 1: Incentives, Contracts and Risk-Sharing

1. **Incentives and Contracts, Lazear (2000):** The largest US installer of automobile glass, Safelite Glass Corporation, moved from hourly wages to piece rate pay in 1994. A study tracked 3,000 employers over 19 months as the piece rates were gradually phased in.

   (a) What are the empirical results?

   (b) **Incentive Effect:** Consider the following simple model: A worker can control how much to produce, but the production is costly. The preference of the worker is $u(t, s|q) = t + sq - c(q)$ where t is the fixed wage component, s is the piece rate, q is the number of windshields produced, $c(q)$ is the cost of producing q windshields. Assume that the cost of producing windshields is convex ($c'(q) > 0$ and $c''(q) > 0$) and worker gets fired if $q < q_{\min}$. What is the optimal decision for the worker? If the outside option gives the utility level $\bar{u}$, which condition should the contract (t, s) satisfy? Now, assume that the worker may not fully control the output due to randomness in the output given the effort exerted. If the worker does not like variations in the outcome, what can you say about the new contract? Is there a trade off or are piece rates unambiguously better then?

   (c) **Selection Effect:** Consider the following simple model: Workers have different ability to produce. A worker with ability q produces q windshields and the production happens at no cost. The preference of the worker is $u(q) = t + sq$. Assume that outside option gives the utility level $\bar{u}$. Given the contract (s, t), what is the minimum ability level for which the worker will work for the employer? What is the difference in Safelite case?

2. **Classifying Asymmetric Information** The following list depicts parties that form a contractual relationship and provides reasons for such contracts. Identify the principal (uninformed party), the agent(s) (informed party), verifiable variables, information asymmetries and information flows.

   (a) Insurance Company, policy holder, genetic disposition for a certain disease.

   (b) Insurance Company, policy holder, care to avoid accidents.

   (c) Environmental Protection Agency, Firm, legal settlement about damage payments for environmental harm.

   (d) Soccer player, club, labor contract.

(e) Buyer, seller, quality or warranty.

(f) Potential buyers, auctioneer, value of the item.

2. Contracting with Complete Information Consider the basic model discussed in chapter 2 of Macho-Stadler and Perez-Castrillo (2001). While in contexts of asymmetric information principals and agents are usually distinguished by their private information, we assume now that the principal offers a contract and the agent can decide wether or not to accept it (take-it-or-leave-it offer).

(a) Explain the optimization program P2.1 and relate its constituents to some of the examples mentioned above (now assuming that there are no information asymmetries).

(b) Consider the efficiency condition (eqn.2.1) and the special case for two states of nature depicted in (eqn.2.3) and try to describe an intuition for this result.

(c) Explain why the participation constraint for the agent will always be binding in any optimal contract proposed by the principal in a take-it-or-leave-it offer.

(d) A contract under complete information is thus fully characterized by (i) the efficiency condition (eqn.2.3) and a binding participation constraint (eqn.2.4). Consider a risk-neutral principal and a risk averse agent. Explain the optimal contract.

(e) Now consider a risk averse principal and a risk neutral agent. Explain the optimal contract and relate it to the previous results.

(f) What changes if both, principal and agent are risk-averse?

(g) As a reaction to the catastrophe in the Indian ocean around Christmas 2004, there was a lively discussion about the debt of less developed countries. A common characteristics of the debt contracts agreed upon by developed and less developed countries is that the risk associated with a fluctuation in exchange-rates is completely taken by the less developed countries (i.e. the credit contracts are specifying transfers in US dollars). Use the insights about risk sharing derived so far to discuss this practice and to draw some conclusions regarding the cancellation of these debt contracts.

(h) In many European countries there is a trend from public insurance (of health or against unemployment) to private insurance. One of the consequences is the pooling of risk (i.e. agreeing on insurance contracts) at several companies instead of pooling in the public budget. Explain, why this might be socially costly. How do insurance companies try to solve the problem of small risk pools?

Problem Set 2: Moral Hazard

1. Production Technologies and Labor Contracts

Consider a principal-agent problem with three exogenous states of nature, θ_1, θ_2 and θ_3; two effort levels, a_L and a_H; and two output levels, distributed as follows as a function of state of nature and the effort level:

State of Nature	θ_1	θ_2	θ_3
Probability	0.25	0.5	0.25
Output under a_H	18	18	1
Output under a_L	18	1	1

The principal is risk neutral, while the agent has utility function $\sqrt{w}$ when receiving monetary compensation w, minus the cost of effort, which is normalized to 0 for a_L and 0.1 for a_H. The agent's reservation expected utility is 0.1.

(a) Derive the first-best contract.

(b) Derive the second-best contract when only output levels are observable.

(c) Assume the principal can buy for a price of 0.1 an information system that allows the parties to verify whether state of nature θ_3 happened or not. Will the principal buy this information system? Discuss.

2. Unemployment Insurance

Consider a worker (Agent) A producing w when employed and 0 when unemployed. Government (Principal) offers A to pay tax τ when A is employed and to receive unemployment benefit b when A is unemployed. The probability of being employed depends on A's effort a, $p(a)$ such that $p(0) = 0$, $p(\infty) = 1$, and $p'(0) > 1$. Exerting effort is also costly and the cost is borne by the agent and the cost of exerting effort a equals a.

(a) The expected utility of the agent who exerts a with a contract (τ, b) equals $p(a)\, u(w - \tau) + (1 - p(a))\, u(b) - a$. Characterize the contract that maximizes the agent's expected utility given an incentive compatibility condition for the worker (first order condition of agent's maximization condition) and balanced budget constraint for the government (principal).

(b) What are the effects of an increase in benefits on utility and effort? (Hint: Using substitution approach replace a and τ by a function of b. Then, solve the unconstrained maximization problem.)

(c) Discuss the direct and indirect effects of an increase in benefits on taxes. (Hint: Differentiate τ with respect to b from the participation constraint (balanced budget constraint for the government). Replace with the elasticity of the probability of unemployment with respect to the unemployment benefit level.)

(d) Find and discuss the condition when the unemployment insurance is optimal (Hint: Rewrite the first order condition with respect to b. As always in economics, optimality is implied by equality of Marginal Benefit and Marginal Cost).

(e) Can we test for the optimality of the current unemployment insurance scheme? (Hint: Meyer (1990) estimates the impact of a change in unemployment benefits on exit rates (probability of finding jobs)) Could there be distortionary effects of unemployment insurance scheme? (for the interested ones: Engen and Gruber (1995), Cullen and Gruber (2000).)

Problem Set 3: Adverse Selection

1. Assume a (risk-neutral) researcher wants to offer a contract to a (risk-neutral) Ph.D. student. The Ph.D. student will work under the direct supervision of the researcher such that effort is observable. There are good students with utility function $U^G(w,e) = w - e^2$ and bad ones $U^B(w,e) = w - 2e^2$ where w is the wage and e is the effort level fixed in the contract. The probability that the student is of type G is $0 < q < 1$. Every student has reservation utility $\underline{\mathrm{U}} = 0$. If the student spends effort e the researcher receives $\Pi(e) = ke$ where k is constant and independent of the students type.

(a) Assume that the researcher has perfect information (i.e. he observes the students type and the type is verifiable). State the principals optimization problem.

(b) Now assume that the researcher only knows q (the distribution of types). Explain why this information asymmetry may lead to a break-down of the market for Ph.D. students if the researcher is only able to offer *one* contract.

(c) Assume that the researcher offers a menu of contracts. Formulate his optimization problem.

(d) Assume that the bad types selection and the good types participation constraint are not binding. Give an economic interpretation of this assumption. How does this simplify the optimization problem?

(e) Show that (given d) the good types selection and the bad types participation constraint are binding.

(f) Which type of student spends an efficient (an inefficient) amount of effort? Explain.

(g) Which type of student earns extra-utility due to asymmetric information? Explain.

(h) Give other (meaningful) examples for this kind of information asymmetry and the respective economic consequences.

2. Price Discrimination by a Monopolist

How can a monopolist maximize its profits if it cannot observe the valuation that consumers have for the good it is selling?

Consider a monopolist producing quantity q of a good at constant marginal cost $MC = c$. There are two types of consumers h-type and l-type. The utility of consuming q of the good and paying T is $u_i(q,T) = \theta_i v(q) - T$ with $\theta_i \in \{\theta_h, \theta_l\}$ and $v' > 0 > v''$. The proportion of l-type is $\theta_l = \beta$, and the proportion of h-type is $\theta_h = 1 - \beta$.

(a) Find the first-best contract when the monopolist can observe which type it is facing.

(b) In the second best, the monopolist cannot observe which type it is facing. The monopolist offers menu of contracts and the agent chooses its preferred contract. The monopolist moves first and *screens* the agents and agent's are not allowed to *signal* their types. Find the second-best contract.

Problem Set 4: Information and Insurance Markets

1. Consider Rothschild and Stiglitz (1976).

(a) Explain Figure 1. How is the line EF constructed? The Figure suggests that EF is always rectangular to the 45 degree line. Does that always hold? Why is the optimal contract located at α^*?

(b) Use Figure 2 to explain why there cannot be a pooling contract in equilibrium.

(c) Consider Figure 3. Explain graphically and intuitively why the high risk type always receives full insurance. Why does the low risk type not receive full insurance in any separating equilibrium?

(d) Discuss the construction of α^L. Why can it not be part of an equilibrium to offer a contract different from α^L (i.e. why not a contract on EL between β and α^L or between α^L and E)?

(e) Use Figure 3 to explain why the proportion of high and low risks is important for the existence of a separating equilibrium.

(f) Consider the case of tests for genetic dispositions. Is a market failure for health insurances likely if theses tests are not feasible? Now, consider the insurance contracts for cars that are contingent on the drivers age. Will the drivers age serve as a proper screening device (and potentially prevent a market breakdown)?

2. Consider a risk-averse individual with utility function of money $u(.)$ and initial wealth W_0 who faces the risk of having an accident and losing an amount x of her wealth. The distribution of x depends on the individual's accident-prevention effort a. Assume the distribution conditional on a is as follows:

$$\begin{aligned}\Pr(x=0|a) &= p(a)\\ \Pr(x=x_L|a) &= (1-p(a))\,\alpha\\ \Pr(x=x_H|a) &= (1-p(a))\,(1-\alpha)\end{aligned}$$

where $x_H > x_L > 0$. Assume the probability function $p(a)$*is increasing and concave in effort*, i.e. $p'(a) > 0 > p''(a)$. The individual's (increasing and convex) cost of effort, separable from her utility of money, is $c(a)$. A risk-neutral principal can offer a contract $R(x)$ of repayments net of the insurance premium when the individual loses x. Determine the first-best and second-best insurance contracts. Discuss.

Problem Set 5: Empirical Tests of Contract Theory

1. **Soccer Leagues as a Natural Experiment** Some authors suggest that the so-called kick-index (a grade for a soccer players performance in a given match) is a coherent measure of performance. (How would you test this hypothesis?) They observe that performance (i.e., the kick-index) decreases in the remaining contract duration of a player (e.g., players with a contract that lasts two more years perform better - on average - than players with a contract for three more years) if one controls for all kinds of observable player characteristics (age, experience etc.).

   - Why is this effect typically taken as an indication of moral hazard?
   - Can you think of alternative explanations?
   - Explain how the different reasons for the correlation between contract duration and performance can be disentangled empirically! Why is the legal change after the Bosman judgment a suitable natural experiment?[7]

2. **Structural Econometric Models and Regulation** Until recently, water and energy supply for private households was organized by local monopolies (Nutsbedrijf, Stadtwerke etc.). As these entities were often profit centers for the respective cities, regulations were designed as to avoid deadweightlosses. One frequently used regulatory design are price-caps (the authority sets a maximum price per unit of the commodity - m^3 water, or kWh electric energy).

   - Explain why it is important for the authority to know whether there is an information asymmetry between the service provider and the consumers (e.g. with respect to the consumers willingness to pay).
   - Suppose you have a dataset with prices and quantities and some observable characteristics of the consumers. How would you try to judge whether there is an information asymmetry?
   - Explain how you would design and estimate a structural model that helps you to identify an information asymmetry! Why is it beneficial for the authority to estimate a structural model?

[7]Before the Bosman Judgement, a player needed the consent of his old club for any transfer even after his contract expired - usually the old club negotiated a transfer fee. The judgment declared these transfer fees illegal.

3. Is the Lab a way out?

- List the most important obstacles to empirical tests of contract theory (and name at least one illustrative example).
- What are the respective remedies proposed by the empirical literature?
- Discuss in how far laboratory experiments offer a useful alternative!

Problem Set 6: Learning

A professor travels to Portugal to teach students about information economics. One evening he is invited over for dinner by one of his students. He is served a juicy T-bone steak. However, the professor knows that 1% of the Portuguese T-bone steaks are contaminated by BSE, and eating from them will surely lead to the disease of Creutzfeld-Jakob. Eating a non-contaminated steak gives a utility equivalent to 10 EUR, but having a contaminated one leads to a utility equivalent to -4000 EUR. Not eating at all gives utility 0. The professor is a utility maximizer with von Neumann-Morgenstern utility function $v(c) = c$.

1. Will the professor eat the steak? The student notices the professors hesitation to eat. The student claims that he has a do-it-yourself test available to check out the condition of the food.

2. Suppose this test works without error. How much would the professor in principle be willing to pay to have the test done?

Unfortunately, even though free from charge, the do-it-yourself test is far from perfect. Nevertheless, the professor insists that the test be done. For non-contaminated steaks the test mistakenly finds contamination in 50% of the cases. What is worse, for contaminated steaks it errs in 10% of the cases.

3. Give a graphic representation of the learning process. What is the probability of a positive test result?

4. What is the posterior probability that a steak is contaminated when the test says so? And what is the posterior probability that a steak is non-contaminated when the test says so?

5. What is the professor willing to pay for this test?

6. How do Kahneman and Tversky's (1971) results relate to Bayesian updating?

7. How does the belief in the law of small numbers influence the value of probabilistic tests (like the one above)?

8. Suppose someone believes in the law of small numbers and participates in an auction for start and landing slots at an airport. He receives an analysts report about the potential value of a certain slot. How does his bidding behavior relate to the bidding behavior of a rational bidder?

Problem Set 7: Signalling

1. Consider the relationship between a producer of a given good and the respective retailer. Assume that the producer has constant marginal costs c and sells a quantity $q = D - p$ where p is the price and D is the demand volume, which can take two values, $D \in \{D^G, D^B\}$ with $D^G > D^B$. The producer offers a franchise contract with transfer payments $T(q) = a + bq$.

   1. Calculate the optimal production decision q, the price and the retailers profit Π for a given contract $T(q)$.

   2. If the demand volume is common knowledge, what is the optimal franchise contract offered by the producer?

   3. Assume from now on that the producer knows the demand parameter D while the retailer does not (until the contract has been signed - he knows about D after the production decision). Show that the producer has incentives to make out that the demand is good, whether this is true or not.

   4. How can the producer signal the retailer the true demand volume?

   5. Assume that the producer offers a menu of contracts $\{(a^G, b^G), (a^B, b^B)\}$. Show which type of contract has higher fixed (variable) payments in a separating equilibrium.

   6. Why does the producer offer the first best contract $((a^B)^\star, (b^B)^\star)$ in any separating equilibrium?

   7. What is the optimal contract (a^G, b^G) in a separating equilibrium that satisfies the intuitive criterion?

2. A simple example: costless signals

Consider N villages $i = 1, 2, ..., N$. Each villager is privately informed of the cost he will incur if he goes hunting with the other villagers. The cost c_i is a priori uniformly distributed on $[0, 1 + \varepsilon]$, where ε is a positive number, and c_i is independently distributed across villagers. If all agree to hunt together, upon capturing a stag they each will get a value 1. However, if just one villager opts to stay home, the others will not be able to catch the stag. The risk for hunter i is that he goes hunting, incurs a cost c_i and gets value of 0 because one of his fellows has preferred to stay at home. Let π be probability that a villager goes hunting. What is the equilibrium of this game? Now, consider a preplay communication with two stages:

- In the first stage each villager announces “yes” or “no” to all others.
- In the second stage, each villager decides whether or not to go hunting.

What is the equilibrium now? If the announcements are replaced with "yellow" or "blue", will there be any difference? Is there any equilibrium in which signals convey no information whatsoever?

3. When the education is productive

Consider a competitive labor market where there are two types of workers, H-type and L-type. It is common knowledge that the fraction ρ of the population is H-type. Worker i-type has the utility function $U_i = w(e) - c_i(e)$, where $w(e)$ is the income as a function of years of education e, and $c_i(e)$ is the cost of acquiring e years of education for i-type. Let $m_i(e)$ be the marginal product of i-type. We assume that for any education level e, $c_H(e) < c_L(e)$ and $m_H(e) > m_L(e)$. Since the education is productive, for each type $m_i(e)$ is not constant but an increasing function e. Assume that $c_H(e) = \frac{1}{2}e^2$, $c_L(e) = \frac{3}{4}e^2$, $m_H(e) = 6e$, and $m_L(e) = 3e$. Also, assume that $\rho = \frac{1}{3}$.

1. Find the full information equilibrium.
2. Let g be the critical level of education. For which values of g, is there a pooling equilibrium?
3. Compute the separating equilibria.

Problem Set 8: Fairness and Contract Design

Consider the set-up in the recent paper by Fehr, Klein, and Schmidt (2007) (FKS, henceforth). Suppose - as in their experiment - that $p = 1/3$ and $f = 13$.

1. Suppose a selfish principal (who beliefs that the agent is selfish) designs an incentive contract. Show that he can not do better than offering a wage $w = 4$ and a fine $f = 13$ (such that the agent chooses an effort $e = 4$).

2. Can the principal improve upon this payoff by offering a trust or bonus contract (suppose again that he and the agent are selfish and common knowledge thereof)? Is this inline with the experimental findings of FKS?

Now consider the model of inequity-aversion as detailed in section 5 of FKS and assume their calibration (fraction of fair agents $q = 0.4$, $\alpha = 2$, and $\beta = 0.6$).

3. Suppose the principal chooses a trust contract.

  (a) Show that a fair principal chooses a contract where pay-offs for him and the agent are equal ($M^A = M^P$).

  (b) Use this to show that effort spend by a fair agent increases in the wage paid to him ($\frac{de}{dw}) > 0$.

  (c) Show that the principals pay-off is decreasing in w if he faces a fraction of $q = 0.4$ fair agents.

  (d) Show that a fair principal pays $w = 5$, which will be accepted by both agents who both choose $e = 1$. The principals expected payoff is $M^P = 5$.

  (e) Show that a selfish principal either offers $w = 0$ (which is accepted by the selfish agent only), or $w = 4$ which is accepted by both types of agents. Show that $M^P = 6$ in both cases.

4. Suppose the principal chooses an incentive contract.

  (a) Show that a selfish principal offers ($w = 4$, $f = 13$) which is accepted by a selfish agent only and yields to an effort level of $e = 4$. Show that the principals pay-off is $M^P = 15.6$.

  (b) Show that a fair principal offers ($w = 17$, $f = 13$) which is accepted by both types of agents and yields to an effort level of $e = 4$ as well. Show that the principals pay-off is $M^P = 13$. Hence, the principal is strictly better off by offering an incentive contract than by offering a trust contract.

5. Suppose the principal chooses a bonus contract.

(a) Show that it is a pooling equilibrium that the principal offers $w = 15$, selfish agents choose $e = 7$ and receive a bonus of 25 from fair principals, while fair agents choose $e = 3$ and receive a bonus of 1 by a fair principal. Selfish principals never pay a bonus.

(b) Show that in this equilibrium the selfish principal gets an expected payoff of $M^P = 39$, and the fair principal a pay-off of $M^P = 23.6$. Hence, the principal is strictly better off by offering a bonus contract than by offering an incentive contract.

6. Can you bring up an intuition for this ranking of contract designs?

7. Use the previous theoretical results to explain the experimental findings of FKS.

8. What is the impact of fairness on social welfare in this case?

Problem Set 9: Auctions

In what follows, we try to analyze the optimal design of an auction (or more generally an allocation mechanism). To this end, we exploit a simple analogy to third-degree price discrimination by a monopolist (for a complete treatment see Bulow and Roberts (1989), third degree price discrimination is covered in any textbook on Industrial Economics).

An auctioneer wants to sell one unit of a commodity to one out of n potential buyers. The auctioneers production costs are zero. Each buyer i values the good with v_i, where vi is distributed with cumulative density $F_i(v_i)$ and private information to i. The support of v_i ($[\underline{v}_i, \bar{v}_i]$) is implicitly given by $F(\underline{v}_i) = 0$ and $F(\bar{v}_i) = 1$.

1. Suppose the good is a production license and two firms are potential buyers. For firm 1 valuations v_1 are uniformly distributed between 0 and 2, for firm 2 valuations v_2 are uniformly distributed between 1 and 2. Draw $F_i(v_i)$ and give an economic interpretation of the two cumulative densities.

2. Demand Function

Suppose for a moment that the auctioneer deals with each potential buyer separately. In order to maximize his revenues, the auctioneer computes monopoly prices in each market segment (i.e., for each potential buyer). Suppose the potential buyer purchases whenever his valuation is above the price. What is the probability that a potential buyer i purchases at price p? Draw (for the densities in 1.) the probability of purchase as a function of price (i.e., the "demand curve").

3. Revenue

With demand as given in 2., compute the revenue of the auctioneer from selling the good to buyer i at price p. Thereby substitute the demand as derived in 2. for the price.

4. Marginal Revenue

Compute marginal revenue and draw it in the graph developed in 2.[8] When designing the optimal allocation mechanism, the seller faces a trade-off similar to the problem of third-degree price discrimination. The seller maximizes his revenue which is the probability of trade (in a given market segment - here, with a certain potential buyer) times benefits from trade within this market segment. The optimal price equates marginal benefits (profit gain from a price increase within a segment) with marginal costs (reduced probability of trade in the segment or likewise increased probability of trade in another segment). Argue why this

[8] For the computation, use the inverse function theorem according to which $\frac{dF^{-1}(x)}{dx} = \frac{1}{\frac{dF(y)}{dy}|_{y=F^{-1}(x)}}$.

implies that a seller who wants to allocate one unit should allocate it to the potential buyer who maximizes marginal revenue.[9]

5. Optimal Allocation
Recall that the auctioneer has only one unit for sale. Suppose that he actually sells to the potential buyer who creates the highest marginal revenue (in case all marginal revenues are negative, he keeps the good - give an economic interpretation of this!). Under which conditions is the allocation of the optimal mechanism (not) efficient? Check the configuration in 1.! Under which conditions is a regular English or Vickrey auction sub- optimal? Give economic illustrations for the alignment and trade-offs between revenue maximization and allocation efficiency (as an example consider the configuration in 1. with v_2 distributed uniformly between 1 and 3 instead).

6. Revelation Mechanism
Suppose the seller wants to allocate optimally (i.e., to the buyer who creates the highest marginal revenue) and he asks potential bidders to submit their valuation. What is the maximal payment you can ask from a potential bidder which induces a dominant strategy to tell the truth? Explain how a mechanism which allocates optimally (see 5.) and induces truthful revelation of valuations looks like. When does this mechanism coincide with a second-price (sealed-bid) auction? Give economic examples for the sub-optimality of second-price (sealed-bid) auctions. How could the optimal mechanism be implemented in reality? Do you know any examples?

Have a look at Milgrom (1989). A bit disappointing for contemporary researchers this old article is still up-to-date from a theoretical point of view. An enormous amount of empirical and experimental studies have, however, supported and questioned the results displayed in Milgrom (1989) in the meantime. Try to answer the following questions with reference to theory (as in Milgrom (1989)) and empirical results that you find in the literature.

1. Where and why do we observe auctions as a trading mechanism?

2. What kind of auction institutions (rules) do you know?

3. Which auction type is most frequently used for what kind of economic environment?

4. What is the main (?) problem in auctions of lost suitcases or art auctions if bidders have asked for expertise beforehand?

[9] The exact derivation of this result can be found in Bulow and Roberts (1989) or in the original paper by Myerson (1981), awarded with the Nobel Prize in 2007. But -unlike the informal discussion here- it can not avoid some integrals.

5. Which auctions are strategically equivalent?

6. Which auctions lead (under certain conditions) to the same allocation and revenues?

7. Which auctions lead to efficient outcomes?

8. Why are first price auctions used in public procurement?

9. Why are English auctions used at Sothebys?

10. Why does no one use the Vickrey (second-price, sealed bid) auction?

11. Explain why the empirical analysis of the allocation of oil-drilling rights in the US from (1954 – 1969) suggests the existence of a winners curse?

C Topics Final Assignment

In the following we provide a list of potential research questions suitable as a starting point for the final assignment. Most of these topics (among others) are discussed in Stiglitz (2000). Other useful sources for the various applications of Information Economics is the recent comprehensive Bolton and Dewatripont, Contract Theory, MIT press, 2005 and Salanié, The Economics of Contracts: A Primer, 2^{nd} Edition, MIT Press, 2005.

1. Asymmetric Information and Labor Markets

   (a) Do wages in professional sports leagues reflect productivity?

   (b) Can incentive contracts for managers be responsible for increasing noise in annual firm reports?

2. Asymmetric Information and Financial Markets

   (a) How can managers benefit from a publication of their purchases of firm shares?

   (b) Why does a financial market provide insufficient credit?

   (c) What is the signalling function of edifices of large banks?

   (d) What are the costs and benefits of micro-credit schemes as promoted for developing countries.

   (e) In Germany, stake- and shareholders often coincide - in contrast to the US. Why (and if - how) does that matter?

3. Regulation

   (a) How can the government aggregate preferences for public goods?

   (b) How can auction design hamper collusion?

   (c) How can health insurance design solve problems of adverse selection and moral hazard?

   (d) What is the social impact of share cropping contracts in developing countries?

   (e) How can know-how transfer be stimulated by incentive contracts?

   (f) How can we design collusion proof mechanisms for procurement auctions?

4. Behavioral aspects of Information Economics

   (a) How could one conduct (out-of laboratory?) tests of the law-of-small-numbers?

(b) How could one test the importance of crowding out of intrinsic motivation for regulatory incentive schemes?

(c) Are limits to information processing an argument in favor or against auction mechanisms (in procurement and sales environments)?

(d) Is there a preference for "truth-telling"?

5. Asymmetric Information in Industrial Organization

(a) Can information asymmetries explain predatory pricing?

(b) Can information asymmetries explain the price differential between first and second class railway tickets or business and economy class for regional flights?

(c) What are the lessons to learn from the 3G telecom license allocation for trade institutions for airport slots or emission certificates?

D Further Literature

In the following, we list papers that might be a useful starting point for the final research project.

D.1 Empirical Contract Theory

1. *How to test for asymmetric information.* Chiappori and Salanie (2000) offer econometric techniques that detect asymmetric information in insurance contract relations. They apply the concepts to the French market for car liability insurance. In particular they discuss methodological shortcomings of preceding contributions.

2. *How to guarantee know-how transfer in a contractual relationship.* This is modelled and tested in Macho-Stadler et al. (1996). Their contribution offers a simple Moral Hazard model that explains certain structures in contracts for technology and know-how transfer. The authors test their theoretical predictions with data about relationships between Spanish and foreign firms.

3. *How to separate Moral Hazard and Adverse Selection.* Feess et al. (2004) develop a simple model for moral hazard and adverse selection to explain contract structures in professional sports leagues. They use legal changes in European soccer to disentangle Moral Hazard and Adverse Selection (Heterogeneity) effects and test their predictions with a dataset about the German Bundesliga.

D.2 Regulations and Incentives

1. *How to regulate a natural monopoly.* In a landmark paper Laffont and Tirole (1986) establish the modern theory of incentive regulation. They apply standard screening techniques to characterize the optimal incentive scheme for a natural monopolist. In particular they explain when the optimal scheme is more similar to cost-plus or fixed-price regulation.

2. *How does regulation work in a political system?* In quite some publications such as Laffont (1996) the so-called Toulouse school of Political Economy (in contrast to the Chicago or Virginia school put forward e.g. by Becker (1985)) discussed regulatory performance in different political systems. The paper describes how public or private ownership of natural monopolies and the preferences of the (electoral) majority influences welfare.

3. *Is Incentive Regulation the Holy Grail?* Recently, papers like Falk and Koesfeld (2006) provided experimental evidence that high-powered incentive schemes may well crowd out (welfare enhancing) intrinsic motivations.

4. *Bypass and Cream Skimming in Regulated Telecommunication.* Laffont and Tirole (1990) investigate the impact of asymmetric information in the regulation of (partly liberalized) telecommunication markets.

D.3 (Strategic) Information Transmission and Processing

1. *What are the limits of information processing?* The seminal paper by Miller (1956) is a compilation of experimental evidence that suggests general limits to our cognitive capacities. A talk should not only review this important contribution but should also briefly mention recent confirmations and qualifications.

2. *Do we believe in the Law of Small Numbers?* Rabin (2000) provides a detailed review of experimental evidence starting with Kahneman and Tversky (1970) that suggest regular patterns of biased information updating. This paper also provides a model framework that allows for a structuring of the empirical findings and for an organized discussion of fields of application such as the market for financial analysts.

3. *The strategic value of advertisement.* Milgrom and Roberts (1986) apply signalling games to explain the advertisement and pricing patterns in markets for experience goods. This paper provides a theory of advertisement without transmission of product related information.

4. *What about just telling the truth?* Social preferences for truth-telling are investigated in Sanchez- Pages and Vorsatz (2005). They elicit social preferences in a simple sender-receiver game, where receivers who detect cheating have the opportunity to punish.

D.4 Market Design and Information Aggregation

1. *What are the lessons to learn from the European auction of 3G telecom licenses?* Many papers such as Klemperer (2002) apply key insights from auction theory to explain the heterogenous outcome of UMTS license auctions all over Europe and derive design suggestions for future rounds.

2. *How to detect collusion in auctions.* Stimulated by Porter and Zona (1993) and the subsequent economic literature, antitrust authorities start to use econometric techniques that provide additional evidence for bid rigging in procurement auctions.

3. *What can economists learn from eBay and vice versa? - Part 1.* Reily (2005) tests some fundamental predictions of auction theory in a field experiment on eBay.

4. *What can economists learn from eBay and vice versa? - Part 2.* Roth and Ockenfels (2002) (and Ockenfels and Roth (2006)) investigate theoretically and empirically the phenomenon of late bidding on the internet.

E Refresher in Expected Utility Theory

This section offers a precise formulation of expected utility theory. Every participant of the course should make himself familiar with the concept of a utility function (i.e. its relation to individual preferences) and should be aware of the conditions when preferences over lotteries can be represented in expected utility form.

E.1 Lotteries

Let C be the set of elementary outcomes. In principle, an elementary outcome may have any nature. In this course, however, we will be mainly concerned with outcomes having the form of monetary payoffs. You can therefore think of elementary outcome c as specifying an amount of money and identify the set C with the set of all (real) numbers. Sometimes we will allow for positive payoffs only, in which case C is the set of positive numbers.

An important notion in the theory of decision making under uncertainty is that of a lottery, also called a prospect or a gamble. A simple lottery p specifies finitely many of uncertain outcomes $c_1, c_2, ..., c_K$ together with the probabilities $p(c_1), p(c_2), ..., p(c_K)$ of each outcome. The probabilities $p(c_k)$ are nonnegative numbers adding up to 1. We denote the set of all simple lotteries by $\mathcal{P}$. A simple lottery p can be thought of as a probability distribution on the set of elementary outcomes C which assigns a positive probability to at most a finite number of outcomes. Let p be a simple lottery which assigns probability $\frac{1}{2}$ to each of the outcomes 0 and 1. Let q be another simple lottery where outcomes -1 and 0 occur with probabilities $\frac{1}{4}$ and $\frac{3}{4}$. Let $(p, q; \alpha, 1-\alpha)$ denote a compound (or complex) lottery in which one receives simple lottery p with probability α and q otherwise. Figure 1 gives a schematic representation of a compound lottery. It should be clear that compound lottery $(p, q; \alpha, 1-\alpha)$ is equivalent to the simple lottery r with outcomes 0, -1, and 1 and probabilities

$$\begin{aligned} r(0) &= \alpha \times 1/2 &+ (1-\alpha) \times 3/4 \\ r(-1) &= \alpha \times 0 &+ (1-\alpha) \times 1/4 \\ r(1) &= \alpha \times 1/2 &+ (1-\alpha) \times 0 \end{aligned}$$

More generally, if p and q are simple lotteries then the compound lottery $(p, q; \alpha, 1-\alpha)$ is equivalent to the simple lottery r which assigns probability $r(c) = \alpha p(c) + (1-\alpha) q(c)$ to every elementary outcome c in the set C.

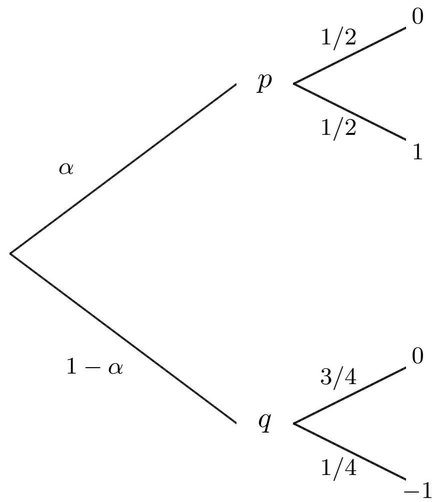

Figure 1: Compound lottery $(p, q; \alpha, 1 - \alpha)$

E.2 Preferences and utility functions

The tastes of the individual with regard to lotteries are summarized by the preference relation $\succeq$ on P. We assume that for any pair p, q of simple lotteries an individual is able to decide whether p is preferred to q (in which case we write $p \succeq q$), q is preferred to p ($q \succeq p$), or he/she is indifferent between p and q ($p \sim q$). Compound lotteries can be compared on the basis of equivalent simple lotteries.

The utility function is another way to describe the tastes of an individual. The utility function U is said to *represent* preference relation $\succeq$ over $\mathcal{P}$ if for any pair of simple lotteries p and q it holds that

$$\text{if } p \succ q \text{ then } U(p) > U(q)$$
$$\text{if } p \sim q \text{ then } U(p) = U(q)$$

One of the central results in microeconomics states that any preference relation satisfying a number of technical and rather unrestrictive assumptions admits a representation by the utility function. Typically, such a representation is not unique. For suppose U is the utility representation of the preference relation $\succeq$. Choose any strictly increasing function f such as logarithmic ($f(x) = \ln(x)$), exponential ($f(x) = e^x$), or linear ($f(x) = ax + b$ where a is a positive number and b is any number). Define function W on the set $\mathcal{P}$ by setting $W(p)$ equal to $f(U(p))$. Function W is then said to be obtained by a monotone transformation of U. It is easy to see that W represents the same preference relation $\succeq$ over the set $\mathcal{P}$. Indeed, if $p \succ q$ then $U(p) > U(q)$ since U is the utility representation of $\succeq$. As f is a strictly increasing function, we have $f(U(p)) > f(U(q))$. So,

W provides a correct ordering of the alternatives p and q. This observation is summarized by the following lemma.

Lemma 1. *Let U be utility function representing the preference relation $\succeq$ on $\mathcal{P}$. Then any function obtained by a monotone transformation of U represents the same preference relation $\succeq$.*

E.3 The expected utility theorem

Definition 1. *The preference relation $\succeq$ satisfies the axiom of complex gambles if for any three simple lotteries p, q, r, and any number $\alpha \in (0,1)$ it holds that*

$$\textit{if } p \succ q \textit{ then } (p, r; \alpha, 1-\alpha) \succ (q, r; \alpha, 1-\alpha)$$
$$\textit{if } p \sim q \textit{ then } (p, r; \alpha, 1-\alpha) \sim (q, r; \alpha, 1-\alpha).$$

Definition 2. *The utility function U on $\mathcal{P}$ has the expected utility form if there exists a function v on C such that the utility of any simple lottery $(c_1, ..., c_K; p)$ is*

$$U(p) = \sum_{k=1}^{K} p(c_k) v(c_k) \tag{1}$$

Equation 1 essentially says that the utility of a lottery p is computed as the mathematical expectation of the values $v(c_1), ..., v(c_K)$. Function v in the above definition is called the preferencescaling function or an elementary utility function. Let us emphasis a crucial distinction between U and v: Function U is defined on the set $\mathcal{P}$ of simple lotteries. Function v is defined on the set C of elementary outcomes.

We proceed to study the relationship between Definitions 1 and 2. In fact, these are equivalent in the following sense: a preference relation $\succeq$ satisfies the axiom of complex gambles if and only if it admits a representation by the utility function of the expected utility form. For convenience, we split this proposition in two parts.

Theorem 1. *Let $\succeq$ be a preference relation on $\mathcal{P}$ represented by the utility function U. Suppose that U has the expected utility form. Then $\succeq$ satisfies the axiom of complex gambles.*

To prove Theorem 1 choose three simple lotteries p, q, and r. If we let t be a simple lottery equivalent to $(p, r; \alpha, 1-\alpha)$ then

$$\begin{aligned} U(t) = & \sum t(c)v(c) \\ = & \sum[\alpha p(c) + (1-\alpha)r(c)]v(c) \\ = & \alpha \sum p(c)v(c) + (1-\alpha)\sum r(c)v(c) \\ = & \alpha U(p) + (1-\alpha)U(r), \end{aligned}$$

where the sums are taken over all those outcomes c that either $p(c)$ or $q(c)$ is positive. Similarly, if w is a simple lottery equivalent to $(q, r; \alpha, 1-\alpha)$ then $U(w) = \alpha U(q) + (1-\alpha)U(r)$. Suppose that $p \succ q$. Then $U(p) > U(q)$, which clearly implies the inequality $U(t) > U(w)$. The compound lottery $(p, r; \alpha, 1-\alpha)$ is therefore preferred to $(q, r; \alpha, 1-\alpha)$, as required by Definition 1. The case $p \sim q$ is dealt with similarly.

Theorem 2. ***The expected utility theorem*** *Let $\succeq$ be a preference relation on $\mathcal{P}$ satisfying the axiom of complex gambles. Then $\succeq$ admits a representation by the utility function of the expected utility form.*

We do not present the proof Theorem 2. The idea is to construct function v using the referencelottery technique.

Example 1. *Suppose that the preference relation $\succeq$ is represented by the utility function*

$$U(p) = \ln[\sum_{k=1}^{K} p(c_k)c_k].$$

Does $\succeq$ satisfy the axiom of complex gambles? Define function W on the set $\mathcal{P}$ by $W(p) = e^{U(p)}$. By Lemma 1, function W is the utility representation of $\succeq$. Moreover, W has the expected utility form, with the function v given by $v(c) = c$. Theorem 2 now implies that $\succeq$ does satisfy the axiom of complex gambles.

E.4 Application: the utility function over actions

In this section we demonstrate that the preference relation over the set of simple lotteries gives rise to the utility function over actions in a natural way. As in Sections 1.11.3 of Hirshleifer and Riley, let $1, ..., S$ be the states of the world, X be a set of actions available to an individual, and suppose that the consequence function $c : (x, s) \mapsto c(x, s)$ is specified. "To choose an act is to choose one of the rows of the consequence matrix like table 1.1 on page 8. Each such row can be regarded as a probability distribution", or a lottery. Thus a lottery associated with action x is $(c_{x1},...,c_{xS};\pi_1,...,\pi_S)$. Now, suppose that the preferences of an individual over the set $\mathcal{P}$ are represented by the utility function U. It is natural to define the utility of action x as that of the associated lottery $(c_{x1},...,c_{xS};\pi_1,...,\pi_S)$. In this way, the preference relation on $\mathcal{P}$ induces the ordering of actions.

In a special case where $\succeq$ satisfies the axiom of complex gambles we can take U to be the function of the expected utility form. The utility of action x can then be computed as

$$U(x) = \sum_{s=1}^{S} \pi_s v(c_{xs}).$$

F Refresher in Risk Aversion

We assume throughout this note that the preference relation of an individual over the set of simple lotteries admit a representation by the expected utility function U. The utility of the lottery p with outcomes $c_1, ..., c_K$ is therefore given by

$$U(p) = \sum_{k=1}^{K} p(c_k)v(c_k). \tag{2}$$

It is often convenient to rewrite equation 2 as

$$U(p) = E_p(v(c)), \tag{3}$$

where the symbol E_p denotes the mathematical expectation under probability distribution p.

Lottery p is said to be nontrivial (or risky) if at least two distinct elementary outcomes occur under p with positive probabilities. That if, a nontrivial lottery involves some uncertainty. We shall say that lottery q is trivial if $q(\bar{c}) = 1$ for some elementary outcome $\bar{c}$ and $q(\bar{c}) = 0$ for any $c \neq \bar{c}$.

Definition 3. *The preferences of the individual are said to exhibit riskaversion if for every non-trivial simple lottery p the following inequality holds:*

$$E_p(v(c)) < v(E_p(c)). \tag{4}$$

To understand the above definition it is crucial to clearly see the difference between the following expressions:

(a) $E_p(v(c))$. This was defined above as a mathematical expectation of the random variable which assumes the values $v(c_1), ..., v(c_k)$.

(b) $E_p(c)$. This is the mathematical expectation of the random variable c under probability distribution p:

$$E_p(c) = \sum_{k=1}^{K} p(c_k)c_k.$$

(c) $v(E_p(c))$. This is the value of function v at the point $E_p(c)$.

The lefthand side of (4) gives the expected utility of lottery p. The righthand side of (4) is the utility of consuming $E_p(c)$ units of income with probability one. Thus, according to Definition 3, an individual is riskaverse if participating in a risky lottery is considered less attractive than receiving the expected income of this lottery with certainty.

Example 2. Let p be a lottery which assigns probability $\frac{1}{2}$ to each of the outcomes 0 and 4. Suppose that $v(c) = \sqrt{c}$. Then

$$\begin{aligned} E_p(v(c)) &= \tfrac{1}{2} \times v(0) + \tfrac{1}{2} \times v(4) &&= 1 \\ E_p(c) &= \tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 4 &&= 2 \\ v(E_p(c)) &= v(2) &&= \sqrt{2}. \end{aligned}$$

Since $\sqrt{2} > 1$, we conclude that the individual in question prefers to receive two units of income with probability 1 than to participate in the lottery where he/she receives 0 or 4 units of income with equal probabilities. Thus, (4) is satisfied, and the individual is indeed risk-averse (in fact, one should verify that (4) holds for every lottery p).

We remark that function v satisfying the condition of Definition 3 is called strictly concave. In the theorem below $v''(c)$ denotes the second derivative of v at point c.

Theorem 3. *Suppose that function v is twice continuously differentiable. Then the agents preferences exhibit risk aversion if and only if $v''(c) < 0$ for any elementary outcome c.*

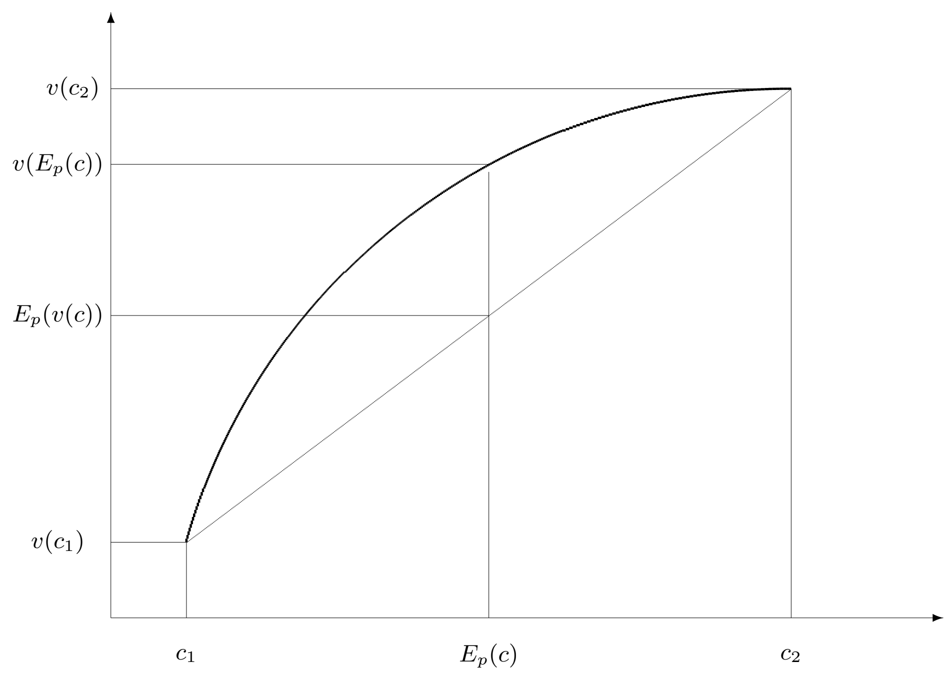

Figure 2: The graph of function v

We now turn to the graphical representation of the risk-aversion hypothesis.

In Figure 2, the horizontal axis measures the elementary outcomes (i.e. payoffs) c, while the vertical axis measures the values of function v. It is the graph of function v. Two possible outcomes of lottery p are indicates as c_1 and c_2. We assume that both of these occur with probability $\frac{1}{2}$, so that the expected payoff $E_p(c)$ of lottery p is located exactly at the midpoint of the interval $[c_1, c_2]$. Correspondingly, the expected utility of p, $E_p(v(c))$, is a midpoint of the interval $[v(c_1), v(c_2)]$. Since the expected utility of p is smaller than that of a certain income $E_p(c)$, we conclude that the individual characterized by function v as depicted in Figure 2 is a risk-averse agent.

G Refresher in Constrained Optimization

A recipe for solving constrained optimization problems

We want to find $x = (x_1, x_2, ..., x_k)$ that solves

$$\max f(x) \quad \text{subject to} \quad g_i(x) \leq c_i \;\text{ for }\; i = 1, 2, ..., n.$$

- Step 1: Form Lagrangian For each of the n constraints, create a multiplier. The multiplier for the constraint $gi(x) \leq ci$ will be denoted by λ_i. Then, the Lagrangian is the function

$$L(x, \lambda) = f(x) - \sum_{i=1}^{n} \lambda_i g_i(x).$$

- Step 2: Write out first–order conditions for x_j's. The first–order condition for the variable x_j for $j = 1, 2, ..., k$ is

$$\frac{\partial L(x, \lambda)}{\partial x_j} = \frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{n} \lambda_i \frac{\partial g_i(x)}{\partial x_j} = 0.$$

- Step 3: Write n constraints. The i-th constraint is

$$g_i(x) \leq c_i.$$

- Step 4: Write the inequality constraints for the multipliers. The multipliers must be all non-negative:

$$\lambda_i \geq 0.$$

- Step 5: Write the complementary slackness conditions. There are n complementary slackness conditions. The ith condition is

$$\lambda_i(c_i - g_i(x)) = 0.$$

- Step 6: Mix all the ingredients. Look for a solution in x and λ to the first-order conditions, the two types of inequality constraints, and the complementary slackness conditions. When and if you find one, it is the solution to your problem.

Problem 1. *A consumer consumes two commodities, wheat and candy. If w is the amount of wheat consumed by this individual and c is the amount of candy, the consumers utility will be given by $u(w, c) = 3\ln(w) + 2\ln(c + 2)$. The individual maximizes his utility subject to the following constraints: (i) the amount of candy and wheat have to be nonnegative, (ii) the individual has ten* *euros to spend and prices are 1 euro per unit of candy or wheat, (iii) a unit of wheat contains 150 calories and a unit of candy 200 calories, and the consumer is constrained to eat no more than 1550 calories.*

1. *Formulate the optimization problem.*
2. *Form the Lagrangian.*
3. *Elicit the first-order conditions.*
4. *Derive the solution.*