Information Economics

Course Manual 1036M
Faculty of Economics and Business Administration
Universiteit Maastricht

Academic Year 2007/2008

1 Introduction

This course deals with the Economics of Information, i.e. with the question of how people decide whenever their information is (in)complete, how they acquire new information, how they learn, and how relationships develop if the different parties have different information about their counterparts and/or the environment. Stated differently, we investigate how an economy adapts to new information, and how this information is disseminated, absorbed and used throughout the economy.

The "Bank of Sweden Award in Economic Sciences in Memory of Alfred Nobel" (there is nothing like a Nobel Prize in economics - we happen to work in a sort of subordinate science) has been awarded to Stiglitz, Akerlof, and Spence in 2001 for their contributions to the Economics of Information. This course will explain why that was well deserved. The fathers of Information Economics are mainly responsible for three contributions that will also serve as a guideline for this course.[1]

- *Information Economics questions standard paradigms of economics.* Information is not just a commodity like many others. If information is distributed unequally over an economy (or a relationship) a lot of paradigms do no longer hold. Market equilibria may not exist, and if they exist they are not necessarily Pareto optimal. Moreover the distribution of income matters, and markets are not necessarily clearing.

- *Information Economics explains empirical puzzles of standard economics.* Taking asymmetric distributions of information as a starting point, many empirical puzzles can be explained. For instance, unobservable management efforts can explain wage differentials that do not reflect different productivity levels (the efficiency wage discussion). The financial structure of a firm can signal the quality of a project (the capital structure puzzle). Or auction may lead to efficient resource allocations while bilateral bargaining does not (a violation of Coase' Theorem). In general: Information Economics helps to explain why market (or institutional) designs matter.

- *Information Economics provides an applicable toolkit to analyze relevant economic settings.* This course will discuss several applications of Information Economics beyond their dad's scope. We will use the toolbox to discuss recent developments in the design of health insurance contracts, electronic marketplaces, , or agreements for technology transfers.

2 Goals

In this course we study the role of information in three dimensions.

- *How is information transmitted in (economic) relations?*

  We investigate different (contractual) relationships with asymmetrically distributed information and analyze the respective (private) benefits and (social) losses. Moreover, we discuss how information is (or can or should be) transmitted in these relationships.

[1] A comprehensive review of the impact of Information Economics on economic thinking can be found in e.g. Stiglitz (2000).

- *How is information processed?*

  If we know how economic relations (or markets) transmit information, an obviously related question is how this information is processed by the respective receiver. This leads to theories of learning and up-dating (and their empirical qualifications from cognitive psychology). Finally, we will discuss how the strategic (abuse) of information influences processing (and transmission).

- *How does market design influence information transmission?*

  If different economic relations (or different institutional arrangements) influence information transmission in the market, it is important to analyze various market designs and their efficiency properties.

The course aims at the provision of a working knowledge in all three dimensions that enables students to identify and analyze problems of information transmission in economic relationships, to evaluate their welfare consequences, and to recommend institutional improvements.

3 Structure

The course contains bi-weekly meetings of two hours each making a total of 14 educational meetings. The first meeting will be spend on an introduction, an overview of basic concepts, and commitments. We continue with 9 Round Table sessions that provide an introduction and a discussion of the relevant theoretical tools. Subsequently, participants will conduct research for their final assignment for one week (instead of the respective meetings, there will be the opportunity to discuss the research with the tutor). In the two final sessions, students will present a draft version of their final assignment. The Schedule is detailed in Appendix A.

3.1 Round Table Sessions

In every Round Table Session a chairperson (allocation of this job will be conducted in the first meeting) organizes a discussion of the assigned reader. Experience shows that it facilitates matters to post questions in advance on the discussion board (of the respective group) such that the chair can organize the different problems. Needless to say that the understanding of the concepts discussed in the reader is absolutely essential in order to understand the literature which is relevant for the final assignment.

3.2 Problem Sets

In each Round Table Session we discuss a Problem Set that provides a useful benchmark for your understanding of the reader. Prepare solutions to the Problem Set that are blackboard-ready. The Problem Sets can be found in Appendix B. Problem Sets marked with $*$ have to be submitted to the tutor before the respective meeting and will be subject to grading.

3.3 Final Assignment

For the final assignment every student has to select a topic that deals with the Economics of Information. A list of potential research questions is enclosed (see Appendix C and - for further inspiration - a list of additional articles in Appendix D) - independent ideas, however, are more than welcome. Experience shows that it pays off to think about the final assignment throughout the course - to discuss the topic with your fellow students and/or the tutor, to conduct a literature research and to organize the paper. Do not postpone everything to the end of the course! You loose plenty of opportunities for discussions, and also the opportunity to let the problem mature. The course schedule leaves space for continuous work on the final assignment - use it!

Research Questions should be claimed on the discussion board (of the course). We apply the usual first-come-first-serve rule. Every topic can only be investigated in one paper. The evaluation criteria for the final assignment are the following formal and content related points:[2]

- The paper can be joined work with one other student. Grading will not discriminate between the contributions of the different authors. If the paper is single authored it must not exceed 17 pages (without front-matter and references), if it is joined it must not exceed 30 pages - double spaced, script 12. Papers not fulfilling these criteria will not pass.

- In the second but last week of the block (Session 11 and 12), you can discuss questions regarding your final assignment with the tutor. In one of these sessions (or - preferably - via email), a table of content for the final assignment - and a plan for the presentation - should be approved by the tutor.

- The paper should deal with a clearly identified research question that can be answered with tools learned in this course.[3] Typical research questions can be found in the Appendix, new (good) research questions are clearly an achievement as such, but should be discussed in advance with the tutor.

- The paper should identify the informational problems of the respective research question and should relate it to the models discussed in the course.

- Discuss the feasibility of empirical test in the respective environment. How would these tests look like? What data-sets would be needed?

- Is there an obvious connection of the research question to experimental or behavioral economics. Will cognitive limits of information processing or non-standard preference structures qualify your results?

- After you assessed the robustness (and the opportunity to check empirical validity) of your findings, you should venture real-life (policy) recommendations.

[2]Of course they do not apply in exactly the same way to all feasible topics.
[3]A successful example will be published on eleum in due course.

The final assignment has to be in the email-box of your tutor until April 4th, 2008, at noon.

3.4 Paper Presentations

Every (group of) student(s) has to present a draft version of their final assignment. A presentation should not last longer than 10 minutes (per person) plus a discussion. The presentation is subject to grading.

The presentation introduces the respective research question, reviews the relevant literature (very briefly), and tries to give a preliminary view on the research for the final assignment. The main goal of this presentation is twofold. First, your fellow students should get an idea about this particular area of information economics (that has not been touched substantially throughout the course). Hence, presenters should make an attempt to introduce the research question and the relevant model(s) starting from insights that are familiar to every participant. Second, the presentation should open up a discussion about the respective research project - with fellow students *and* the tutor. Presenters should therefore try to stimulate feedback and critical comments.

4 Grading Policies

Your evaluation will be based on four parts: your participation in the group (30%), your presentation (10%), the final assignment (30%), and the submitted problem sets (30%). An insufficient mark for any of these parts cannot be compensated. As a result, the passing requirement is 5.5 for each part separately.

The evaluation of your participation will be based on: your presence, both physically and mentally, which is expected in every session; your contributions in raising meaningful questions; your contributions to bringing out problems; your contributions to solving problems of interpretation (of yourself and others) and the Problem Sets; and your performance as chairperson. A trend for the participation grade will be published after the first half of the course. Presentations will be graded directly after the respective session. The mark for the final assignment and the overall grade are published on eleum.

You are expected to be present in every meeting. If you cannot attend a certain meeting for a serious reason, please contact the block coordinator. If you miss a Round Table Session you have to hand-in a fully worked-out solution of the *entire* Problem Set that was scheduled for the respective meeting before the next session.

5 Literature list

The core reference for this course is the comprehensive:

- Macho-Stadler, I. and D. Perez-Castrillo, An Introduction to the Economics of Information, 2nd edition, Oxford University Press 2001.

In addition to Macho-Stadler and Perez-Castrillo (2001) we will provide all papers mentioned in the reference list below as downloads in the section Course Material (Readings) of the eleum site of this course.

For assumed pre-existing knowledge on microeconomics we refer to your textbook(s) on microeconomics. In Appendix E and F, you will find a brief refresher in expected utility theory and the theory of risk aversion. We will assume knowledge of these concepts throughout the course.

In particular, all course participants are asked to check their mathematical skills on the exercise in Appendix G. The mathematics of constrained optimization is a necessary pre-requisite for the entire course. A detailed solution to the exercise with exhaustive explanations can be found in Appendix A1.2 in Kreps, D., A Course in Microeconomic Theory, Harvester Wheatsheaf.

6 The planning group

The planning group consists of

- Markus Walzl (e-mail: m.walzl@algec.unimaas.nl, phone: 83807, office: A 1.09 (TS53))
- Cagatay Kayi (e-mail: c.kayi@algec.unimaas.nl, phone: 83703, office: A 1.020 (TS49)).

The course will be coordinated by Markus Walzl and tutorial meetings are conducted by Cagatay Kayi.

7 References

Becker, G. (1983), A Theory of Competition Among Pressure Groups For Political Influence, Quaterly Journal of Economics, 98(3), 371-400.

Bulow, J. and J. Roberts (1989), The Simple Economics of Optimal Auctions, Journal of Political Economy, 97(5), 1060-1090.

Chiappori, P.A. and B. Salanie (2000), Testing for Asymmetric Information in Insurance Markets, Journal of Political Economy 108(1), 56-78.

Chiappori, P.A. and B. Salanie (2002), Testing Contract Theory: A Survey of Some Recent Work, Lecture delivered at the World Congress of the Econometric Society in Seattle.

Falk, A. and M. Kosfeld (2005), The Hidden Costs of Control, American Economic Review, forthcoming.

Feess, E., B. Frick, and G. Muehlheusser (2005), Legal Restrictions on Buyout Fees: Theory and Evidence From German Soccer, Technical University of Aachen Discussion Paper.

Fehr, E., A. Klein, and K. Schmidt (2007), Fairness and Contract Design, Econometrica 75(1), 121-154.

Kahneman, D. and A. Tversky (1971), Belief in the Law of Small Numbers, Psychological Bulletin 76(2), 105-110.

Klemperer, P. (2002), How (Not) to Run Auctions: The European 3G Telecom Auctions, European Economic Review 46(4-5), 829-845.

Klemperer, P. (2005), What really matters in Auction Design, Journal of Economic perspectives, forthcoming.

Laffont, J.J. and J. Tirole (1986), Using Cost Observations to Regulate Firms, Journal of Political Economy 94(3), 614-641.

Laffont, J.J. and J. Tirole (1990), Optimal Bypass and Cream-Skimming, American Economic Review 80(5), 1042-61.

Laffont, J.J. (1996), Industrial Policy and Politics, International Journal of Industrial Organization 14, 1-27.

Macho-Stadler, I., X. Martinez-Giralt, and J.D. Perez-Castrillo (1996), The Role of Information in Licensing Contract Design, Research Policy 25, 43-57.

Milgrom, P. and J. Roberts (1986), Price and Advertising Signals of Product Quality, Journal of Political Economy 94(4), 796-821.

Milgrom, P. (1989), Auctions and Bidding: A Primer, Journal of Economic Perspectives 3(3), 3-22.

Miller, G.A. (1956), The Magical Number Seven Plus or Minus Two: Some Limits on Our Capacity for Processing Information, Psychological Review 63, 81-97.

Ockenfels, A. and A. Roth (2005), Late Bidding in Second Price Internet Auctions: Theory and Evidence Concerning Different Rules for Ending an Auction., Games and Economic Behavior, forthcoming.

Porter, R.H. and J.D. Zona (1993), Detection of Bid Rigging in procurement Auctions, Journal of Political Economy 101(3), 518-538.

Rabin, M. (2000), Inference by Believers in the Law of Small Numbers, University of California at Berkeley Discussion Paper E00'282.

Reilly, D. (2005), Field Experiments on the Effects of Reserve Prices in Auctions: More Magic on the Internet, University of Arizona Discussion Paper.

Roth, A. and A. Ockenfels (2002), Last-Minute Bidding and the Rules for Ending Second Price Auctions: Evidence from eBay and Amazon Auctions on the Internet, American Economic Review 92(4), 1093-1103.

Rothschild, M. and J. Stiglitz (1976), Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, Quarterly Journal of Economics 40, 629-649.

Sanchez-Pages, S. and M. Vorsatz (2005), An Experimental Study of Truth-Telling in a Sender-Receiver Game, Maastricht University Discussion Paper.

Stiglitz, J. (2000), The Contribution of the Economics of Information to Twentieth Century Economics, Quarterly Journal of Economics 115(4), 1441-1478.

van Zandt, T. (2002), Introduction to the Economics of Uncertainty and Information, INSEAD.

A Schedule

1. Block Opening: Refresher in Choice under Uncertainty

   - Overview and Technicalities
   - Throughout this session we will review the essence of the theory of choice under uncertainty which is the basic toolkit for Information Economics. A preparatory look at the respective reader will do no harm. For students with a bachelor in Econ the material should be familiar, for students with a different background we recommend a closer look. In particular, students are asked to solve the exercise in Appendix G. It will be impossible to follow this course without mastery of the calculus of constrained optimization.
   - Reader: Appendix E, F and G.

2. Round Table 1: Contracts and Risk-Sharing

   - As a benchmark, this session deals with the simplest contractual relationship - an agreement on risk-sharing under complete information. We will analyze optimal risk-sharing and apply our findings to some real-life settings.
   - Reader: Macho-Stadler and Perez-Castrillo (2001) (henceforth, MSPC) ch. 1. and ch. 2.
   - Problem Set 1

3. Round Table 2: Moral Hazard

   - This session introduces the standard model for contracts with hidden actions (moral hazard). We analyze optimal contract structures and welfare implications if actions are not observable and apply our findings to performance dependent wage agreements.
   - Reader: MSPC ch. 3.1-3.3
   - Problem Set 2* (to be submitted for grading)

4. Round Table 3: Adverse Selection

   - This session introduces the standard model for contracts with hidden characteristics (adverse selection). We analyze optimal contracts and welfare implications if characteristics are not observable and apply our findings to monopolistic price discrimination.
   - Reader: MSPC ch. 4.1 and 4.2
   - Problem Set 3

5. Round Table 4: Information and Insurance Markets

   - No course without a ”Nobel-prize” paper. This session provides a detailed discussion of one of the first papers in Information Economics: The insurance market model by Rothschild and Stiglitz (1976). We will use this model to identify conditions for market failures due to asymmetric information.

- Reader: Rothschild and Stiglitz (1976)
- Problem Set 4

6. Round Table 5: Empirical Tests of Contract Theory

    - In this session, we deal with the recent literature on empirical tests of contract theory. Hereby, we will focus on the detection and relevance of asymmetric information effects in insurance, labor, and technology transfer markets.
    - Reader: Chiappori and Salanie (2002)p.1-43 (except sections 3.2 - 3.4).
    - Problem Set 5

7. Round Table 6: Learning and Information Processing

    - We continue with an introduction to the theory of rational learning behavior (i.e Bayesian updating) and apply the knowledge to an analysis of probabilistic test designs. Furthermore, we discuss some classics in cognitive psychology that illustrate the limits of rational learning behavior.
    - Reader: van Zandt ch. 2.1 and 2.2, Kahnemann and Tversky (1971)
    - Problem Set 6* (to be submitted for grading)

8. Round Table 7: Strategic Information Transmission (Signalling)

    - Endowed with a theory about learning, we are ready to address the strategic (ab)use of information in signalling games. As a key example, we consider Spence' job market model with education as a signal.
    - Reader: MSPC ch. 5.1-5.4
    - Problem Set 7

9. Round Table 8: Fairness and Contract Design

    - In this session we discuss additional insights into incentive and signaling theory provided by laboratory experiments. In a recent paper Fehr, Klein, and Schmidt (2007) show how principals design contracts in laboratory experiments and thereby anticipate social preference structures.
    - Reader: Fehr, Klein, and Schmidt (2007).
    - Problem Set 8* (to be submitted for grading)

10. Round Table 9: Auctions

    - We conclude the round table sessions with an analysis of auctions, one of the most popular trade institutions in our days - and one of the most ideal playgrounds for economists.

- Reader: Milgrom (1989)
- Problem Set 9

11. Paper Consultation Session 1
12. Paper Consultation Session 2
13. Presentation Session 1[4]
14. Presentation Session 2

[4] Presentation Schedule will be published on eleum in due course. Obviously, there will be no session scheduled for Easter Monday. If necessary, the tutor will schedule a second presentation session throughout the last week of the block.

B Problem Sets

Problem Set 1: Basic Concepts and Optimal Risk Sharing Contracts

1. Classifying Asymmetric Information

The following list depicts parties that form a contractual relationship and provides reasons for such contracts. Identify the principal (uninformed party), the agent(s) (informed party), verifiable variables, information asymmetries and information flows.

1. Insurance Company, policy holder, genetic disposition for a certain decease.
2. Insurance Company, policy holder, care to avoid accidents.
3. Environmental Protection Agency, Firm, legal settlement about damage payments for environmental harm.
4. worker, employer, labor contract.
5. soccer player, club, labor contract.
6. licensor, licensee, quality of the technology.
7. buyer, seller, quality or warranty.
8. potential buyers, auctioneer, value of the item.

2. Contracting with Complete Information

Consider the basic model discussed in chapter 2 of Macho-Stadler and Perez-Castrillo (2001). While in contexts of asymmetric information principals and agents are usually distinguished by their private information (see above), we assume now that the principal offers a contract and the agent can decide wether or not to accept it (take-it-or-leave-it offer).

1. Explain the optimization program $P^{2.1}$ and relate its constituents to some of the examples mentioned above (now assuming that there are no information asymmetries).
2. Consider the efficiency condition (eqn. 2.1) and the special case for two states of nature depicted in eqn. (2.3) and try to describe an intuition for this result.
3. Explain why the participation constraint for the agent will always be binding in any optimal contract proposed by the principal in a take-it-or-leave-it offer.
4. A contract under complete information is thus fully characterized by (i) the efficiency condition (eqn. 2.3) and a binding participation constraint (eqn. 2.4). Consider a risk-neutral principal and a risk avers agent. Explain the optimal contract.
5. Now consider a risk avers principal and a risk neutral agent. Explain the optimal contract and relate it to the previous results.

6. What changes if both, principal and agent are risk-avers?
7. As a reaction to the catastrophe in the Indian ocean around Christmas 2004, there was a lively discussion about the debt of less developed countries. A common characteristics of the debt contracts agreed upon by developed and less developed countries is that the risk associated with a fluctuation in exchange-rates is completely taken by the less developed countries (i.e. the credit contracts are specifying transfers in US dollars). Use the insights about risk sharing derived so far to discuss this practice and to draw some conclusions regarding the cancellation of these debt contracts.
8. In many European countries there is a trend from public insurance (of health or against unemployment) to private insurance. One of the consequences is the pooling of risk (i.e. agreeing on insurance contracts) at several companies instead of pooling in the public budget. Explain, why this might be socially costly. How do insurance companies try to solve the problem of small risk pools?

Problem Set 2: Moral Hazard

1. Production Technologies and Labor Contracts

A risk-neutral principal can hire a worker who can exert two different effort levels - $e \in \{\text{good,bad}\}$ - which induce a production error with probability $1/4$ and $3/4$, respectively. The workers utility function is $U(w, e) = 100 - (10/w) - v(e)$, where w is the wage received and $v(e = \text{good}) = 2$ and $v(e) = 0$ otherwise. Production errors are verifiable. Without production error the worker produces a good that is worth 20 and worthless otherwise. The worker's reservation utility is normalized to zero.[5]

1. Assume that the workers effort is verifiable. What is the optimal contract offer if the principal wants to implement a low (high) effort level?
2. Which effort level will be implemented by the principal?
3. Assume that the workers effort is non-verifiable from now on. What is the optimal contract offer if the principal wants to implement a low (high) effort level?
4. Which effort level will be implemented by the principal?

2. Financing R& D

Consider a research project with investment costs $C(e) = e$ that will result in a new successful product development with probability $p(e)$ (worth $R > 0$ for the firm) (with $p'(e) > 0, p''(e) < 0$ and $\lim_{e \to \infty} p(e) = 1$).

1. Explain the assumptions on the research technology ($p(e)$).
2. What is the firms optimal effort level?
3. What are potential reasons for subsidies of R& D by the government? Are there counter-arguments? How can the government implement a first best if effort is verifiable?
4. Assume that the firms effort level is not verifiable from now on. What is the effect of a subsidy given as a fixed payment before the firm starts the research activity?
5. What is the result of a research award for successful projects?
6. Assume that a successful research project creates a value of $S > 0$ for the society. Derive a subsidy that implements a first best effort level. Explain the situation for risk avers firms.
7. Explain the connection between the first best subsidy and a patent.
8. What are drawbacks (in terms of social welfare) of a patent system? Can you think of an improvement?

[5] See also Exercise 2 in chapter 3 of Macho-Stadler and Perez-Castrillo (2001).

Problem Set 3: Adverse Selection

Assume a (risk-neutral) researcher wants to offer a contract to a (risk-neutral) phD student. The phD student will work under the direct supervision of the researcher such that effort is observable. There are good students with utility function $U^G(w,e) = w - e^2$ and bad ones $U^B(w,e) = w - 2e^2$ where w is the wage and e is the effort level fixed in the contract. The probability that the student is of type G is $0 < q < 1$. Every student has reservation utility $\underline{U} = 0$. If the student spends effort e the researcher receives $\Pi(e) = ke$ where k is constant and independent of the student's type.

1. Assume that the researcher has perfect information (i.e. he observes the students type and the type is verifiable). State the Principal's optimization problem.

2. Now assume that the researcher only knows q (the distribution of types). Explain why this information asymmetry may lead to a break-down of the market for phD students if the researcher is only able to offer *one* contract.

3. Assume that the researcher offers a menu of contracts. Formulate his optimization problem.

4. Assume that the bad type's selection and the good type's participation constraint are not binding. Give an economic interpretation of this assumption. How does this simplify the optimization problem?

5. Show that (given 4.) the good type's selection and the bad type's participation constraint are binding.

6. Which type of student spends an efficient (an inefficient) amount of effort? Explain.

7. Which type of student earns extra-utility due to asymmetric information? Explain.

8. Give other (meaningful) examples for this kind of information asymmetry and the respective economic consequences.

Problem Set 4: Information and Insurance Markets

Consider Rothschild and Stiglitz (1976).

1. Explain Figure 1. How is the line EF constructed? The Figure suggests that EF is always rectangular to the 45 degree line. Does that always hold? Why is the optimal contract located at α^*.

2. Use Figure 2 to explain why there cannot be a pooling contract in equilibrium.

3. Consider Figure 3. Explain graphically and intuitively why the high risk type always receives full insurance. Why does the low risk type not receive full insurance in any separating equilibrium?

4. Discuss the construction of α^L. Why can it not be part of an equilibrium to offer a contract different from α^L (i.e. why not a contract on EL between β and α^L or between α^L and E)?

5. Use Figure 3 to explain why the proportion of high and low risks is important for the existence of a separating equilibrium.

6. Consider the case of tests for genetic dispositions. Is a market failure for health insurances likely if theses tests are not feasible? Would the feasibility of such tests establish a functioning market?

7. Compare your results to the situation of insurance contracts for cars that are contingent on the driver's age. Will the driver's age serve as a proper screening device (and potentially prevent a market breakdown)?

8. Name other markets with a low or high probability of a market-breakdown.

Problem Set 5: Empirical Tests of Contract Theory

1. A Recent UM Example Consider Walters' article in the NRC Handelsblad (01-20-2007)[6] about educational research that has been recently carried out at UM. What are the conclusions drawn by the author(s)? Discuss these conclusions with respect to unobserved heterogeneity, endogenous matching, and causality (as discussed in Chappori and Salanie (2003) - e.g. on p. 2 and 3).

2. Soccer Leagues as a Natural Experiment Some Authors suggest that the so-called kick-index (a grade for a soccer player's performance in a given match) is a coherent measure of performance. (How would you test this hypothesis?) They observe that performance (*i.e.,* the kick-index) decreases in the remaining contract duration of a player (*e.g.,* players with a contract that lasts two more years perform better - on average - than players with a contract for three more years) if one controls for all kinds of observable player characteristics (age, experience etc.).

- Why is this effect typically taken as an indication of moral hazard?
- Can you think of alternative explanations?
- Explain how the different reasons for the correlation between contract duration and performance can be disentangled empirically! Why is the legal change after the Bosman judgment a suitable natural experiment?[7]

3. Structural Econometric Models and Regulation Until recently, water and energy supply for private households was organized by local monopolies (Nutsbedrijf, Stadtwerke etc.). As these entities were often profit centers for the respective cities, regulations were designed as to avoid deadweight-losses. One frequently used regulatory design are price-caps (the authority sets a maximum price per unit of the commodity - m^3 water, or kWh electric energy).

- Explain why it is important for the authority to know wether there is an information asymmetry between the service provider and the consumers (e.g. with respect to the consumer's willingness to pay).
- Suppose you have a dataset with prices and quantities and some observable characteristics of the consumers. How would you try to judge wether there is an information asymmetry?
- Explain how you would design and estimate a structural model that helps you to identify an information asymmetry! Why is it beneficial for the authority to estimate a structural model?

[6] The article is in Dutch. Please find coorperative solutions to deal with this problem.

[7] Before the Bosman Judgement, a player needed the consent of his old club for any transfer even after his contract expired - usually the old club negotiated a transfer fee. The judgment declared these transfer fees illegal.

4. Is the Lab a way out?

- List the most important obstacles to empirical tests of contract theory (and name at least one illustrative example).
- What are the respective remedies proposed by the empirical literature?
- Discuss in how far laboratory experiments offer a useful alternative!

Problem Set 6: Learning

A professor travels to Portugal to teach students about information economics. One evening he is invited over for dinner by one of his students. He is served a juicy T-bone steak. However, the professor knows that 1% of the Portugese T-bone steaks are contaminated by BSE, and eating from them will surely lead to the disease of Creutzfeld-Jakob. Eating a non-contaminated steak gives a utility equivalent to 10 EUR, but having a contaminated one leads to a utility equivalent to -4000 EUR. Not eating at all gives utility 0. The professor is a utilty maximizer with vonNeumann-Morgenstern utility function $v(c) = c$.

1. Will the professor eat the steak?

The student notices the professor's hesitation to eat. The student claims that he has a do-it-yourself test available to check out the condition of the food.

2. Suppose this test works without error. How much would the professor in principle be willing to pay to have the test done?

Unfortunately, even though free from charge, the do-it-yourself test is far from perfect. Nevertheless, the professor insists that the test be done. For non-contaminated steaks the test mistakenly finds contamination in 50% of the cases. What is worse, for contaminated steaks it errs in 10% of the cases.

3. Give a graphic representation of the learning process. What is the probability of a positive test result?

4. What is the posterior probability that a steak is contaminated when the test says so? And what is the posterior probability that a steak is non-contaminated when the test says so?

5. What is the professor willing to pay for this test?

6. How do Kahneman and Tversky (1971)'s results relate to Bayesian up-dating?

7. How does the belief in the law of small numbers influence the value of probabilistic tests (like the one above).

8. Suppose someone believes in the law of small numbers and participates in an auction for start and landing slots at an airport. He receives an analyst's report about the potential value of a certain slot. How does his bidding behavior relate to the bidding behavior of a rational bidder?

Problem Set 7: Signalling

Consider the relationship between a producer of a given good and the respective retailer. Assume that the producer has constant marginal costs c and sells a quantity $q = D - p$ where p is the price and D is the demand volume, which can take two values, $D \in \{D^G, D^B\}$ with $D^G > D^B$. The producer offers a franchise contract with transfer payments $T(q) = a + bq$.[8]

1. Calculate the optimal production decision q, the price and the retailer's profit Π for a given contract $T(q)$.
2. If the demand volume is common knowledge, what is the optimal franchise contract offered by the producer?
3. Assume from now on that the producer knows the demand parameter D while the retailer does not (until the contract has been signed - he knows about D after the production decision). Show that the producer has incentives to make out that the demand is good, wether this is true or not.
4. How can the producer signal the retailer the true demand volume?
5. Assume that the producer offers a menu of contracts $\{(a^G, b^G), (a^B, b^B)\}$. Show which type of contract has higher fixed (variable) payments in a separating equilibrium.
6. Why does the producer offer the first best contract $((a^B)^*, (b^B)^*)$ in any separating equilibrium.
7. What is the optimal contract (a^G, b^G) in a separating equilibrium that satisfies the intuitive criterion?

[8] See also Exercise 5 in chapter 5 of Macho-Stadler and Perez-Castrillo (2001).

Problem Set 8: Fairness and Contract Design

Consider the set-up in the recent paper by Fehr, Klein, and Schmidt (2007) (FKS, henceforth). Suppose - as in their experiment - that $p = 1/3$ and $\overline{f} = 13$.

1. Suppose a selfish principal (who beliefs that the agent is selfish) designs an incentive contract. Show that he can not do better than offering a wage $w = 4$ and a fine $f = 13$ (such that the agent chooses an effort $e = 4$).
2. Can the principal improve upon this payoff by offering a trust or bonus contract (suppose again that he and the agent are selfish and common knowledge thereof)? Is this inline with the experimental findings of FKS?

Now consider the model of inequity-aversion as detailed in section 5 of FKS and assume their calibration (fraction of fair agents $q = 0.4$, $\alpha = 2$, and $\beta = 0.6$).

1. Suppose the principal chooses a trust contract.
   (a) Show that a fair principal chooses a contract where pay-offs for him and the agent are equal ($M^A = M^P$).
   (b) Use this to show that effort spend by a fair agent increases in the wage paid to him ($\frac{de}{dw}$) > 0.
   (c) Show that the principal's pay-off is decreasing in w if he faces a fraction of $q = 0.4$ fair agents.
   (d) Show that a fair principal pays $w = 5$, which will be accepted by both agents who both choose $e = 1$. The principal's expected payoff is $M^P = 5$.
   (e) Show that a selfish principal either offers $w = 0$ (which is accepted by the selfish agent only), or $w = 4$ which is accepted by both types of agents. Show that $M^P = 6$ in both cases.
2. Suppose the principal chooses an incentive contract.
   (a) Show that a selfish principal offers $(w = 4, f = 13)$ which is accepted by a selfish agent only and yields to an effort level of $e = 4$. Show that the principal's pay-off is $M^P = 15.6$.
   (b) Show that a fair principal offers $(w = 17, f = 13)$ which is accepted by both types of agents and yields to an effort level of $e = 4$ as well. Show that the principal's pay-off is $M^P = 13$. Hence, the principal is strictly better off by offering an incentive contract than by offering a trust contract.
3. Suppose the principal chooses a bonus contract.

(a) Show that it is a pooling equilibrium that the principal offers $w = 15$, selfish agents choose $e = 7$ and receive a bonus of 25 from fair principals, while fair agents choose $e = 3$ and receive a bonus of 1 by a fair principal. Selfish principals never pay a bonus.

(b) Show that in this equilibrium the selfish principal gets an expected payoff of $M^P = 39$, and the fair principal a pay-off of $M^P = 23.6$. Hence, the principal is strictly better off by offering a bonus contract than by offering an incentive contract.

4. Can you bring up an intuition for this ranking of contract designs?

5. Use the previous theoretical results to explain the experimental findings of FKS.

6. What is the impact of fairness on social welfare in this case?

Problem Set 9: Auctions

In what follows, we try to analyze the optimal design of an auction (or more generally an allocation mechanism). To this end, we exploit a simple analogy to third-degree price discrimination by a monopolist (for a complete treatment see Bulow and Roberts (1989), third degree price discrimination is covered in any textbook on Industrial Economics).

An auctioneer wants to sell one unit of a commodity to one out of n potential buyers. The auctioneer's production costs are zero. Each buyer i values the good with v_i, where v_i is distributed with cumulative density $F_i(v_i)$ and private information to i. The support of v_i ($[\underline{v}_i, \overline{v}_i]$) is implicitly given by $F(\underline{v}_i) = 0$ and $F(\overline{v}_i) = 1$.

1. Suppose the good is a production license and two firms are potential buyers. For firm 1 valuations v_1 are uniformly distributed between 0 and 2, for firm 2 valuations v_2 are uniformly distributed between 1 and 2. Draw $F_i(v_i)$ and give an economic interpretation of the two cumulative densities.

2. *Demand Function*
   Suppose for a moment that the auctioneer deals with each potential buyer separately. In order to maximize his revenues, the auctioneer computes monopoly prices in each market segment (i.e., for each potential buyer). Suppose the potential buyer purchases whenever his valuation is above the price. What is the probability that a potential buyer i purchases at price p. Draw (for the densities in 1.) the probability of purchase as a function of price (i.e., the "demand curve").

3. *Revenue*
   With demand as given in (2), compute the revenue of the auctioneer from selling the good to buyer i at price p. Thereby substitute the demand as derived in (2.) for the price.

4. *Marginal Revenue*
   Compute marginal revenue and draw it in the graph developed in (2.).[9] When designing the optimal allocation mechanism, the seller faces a trade-off similar to the problem of third-degree price discrimination. The seller maximizes his revenue which is the probability of trade (in a given market segment - here, with a certain potential buyer) times benefits from trade within this market segment. The optimal price equates marginal benefits (profit gain from a price increase *within* a segment) with marginal costs (reduced probability of trade in the segment or likewise increased probability of trade in another segment). Argue why this implies that a seller who wants to allocate one unit should allocate it to the potential buyer who maximizes marginal revenue.[10]

[9] For the computation, use the inverse function theorem according to which $\frac{dF^{-1}(x)}{dx} = \frac{1}{\frac{dF(y)}{dy}|_{y=F^{-1}(x)}}$.

[10] The exact derivation of this result can be found in Bulow and Roberts (1989) or in the original paper by Myerson (1981), awarded with the Nobel Price in 2007. But - unlike the informal discussion here - it can not avoid some integrals.

5. *Optimal Allocation*
Recall that the auctioneer has only one unit for sale. Suppose that he actually sells to the potential buyer who creates the highest marginal revenue (in case all marginal revenues are negative, he keeps the good - give an economic interpretation of this!). Under which conditions is the allocation of the optimal mechanism (not) efficient? Check the configuration in (1.)! Under which conditions is a regular english or Vickrey auction sub- optimal? Give economic illustrations for the alignment and trade-offs between revenue maximization and allocative efficiency (as an example consider the configuration in (1.) with v_2 distributed uniformly between 1 and 3 instead).

6. *Revelation Mechanism*
Suppose the seller wants to allocate optimally (i.e., to the buyer who creates the highest marginal revenue) and he asks potential bidders to submit their valuation. What is the maximal payment you can ask from a potential bidder which induces a dominant strategy to tell the truth? Explain how a mechanism which allocates optimally (see 5.) and induces truthful revelation of valuations looks like. When does this mechanism coincide with a second-price (sealed-bid) auction? Give economic examples for the sub-optimality of second-price (sealed-bid) auctions. How could the optimal mechanism be implemented in reality? Do you know any examples?

Have a look at Milgrom (1989). A bit disappointing for contemporary researchers this old article is still up-to-date from a theoretical point of view. An enormous amount of empirical and experimental studies have, however, supported and questioned the results displayed in Milgrom (1989) in the meantime. Try to answer the following questions with reference to theory (as in Milgrom (1989)) *and* empirical results that you find in the literature.

1. Where and why do we observe auctions as a trading mechanism?

2. What kind of auction institutions (rules) do you know?

3. Which auction type is most frequently used for what kind of economic environment?

4. What is the main (?) problem in auctions of lost suitcases or art auctions if bidders have asked for expertise beforehand?

5. Which auctions are strategically equivalent?

6. Which auctions lead (under certain conditions) to the same allocation and revenues?

7. Which auctions lead to efficient outcomes?

8. Why are first price auctions used in public procurement?

9. Why are English auctions used at Sotheby's?

10. Why does no one use the Vickrey (second-price, sealed bid) auction?
11. Explain why the empirical analysis of the allocation of oil-drilling rights in the US from (1954-69) suggests the existence of a winner's curse?

C Topics Final Assignment

In the following we provide a list of potential research questions suitable as a starting point for the final assignment. Most of these topics (among others) are discussed in Stiglitz (2000). Another useful source for the various applications of Information Economics is the recent comprehensive Bolton and Dewatripont, Contract Theory, MIT press, 2004.

1. Asymmetric Information and Labor Markets

   (a) Do wages in professional sports leagues reflect productivity?

   (b) Can incentive contracts for managers be responsible for increasing ”noise” in annual firm reports?

2. Asymmetric Information and Financial Markets

   (a) How can managers benefit from a publication of their purchases of firm shares?

   (b) Why does a financial market provide insufficient credit?

   (c) What is the signalling function of edifices of large banks?

   (d) What are the costs and benefits of micro-credit schemes as promoted for developing countries.

   (e) In Germany, stake- and shareholders often coincide - in contrast to the US. Why (and if - how) does that matter?

3. Regulation

   (a) How can the government aggregate preferences for public goods?

   (b) How can auction design hamper collusion?

   (c) How can health insurance design solve problems of adverse selection and moral hazard?

   (d) What is the social impact of share cropping contracts in developing countries?

   (e) How can know-how transfer be stimulated by incentive contracts?

   (f) How can we design collusion proof mechanisms for procurement auctions?

4. Behavioral aspects of Information Economics

   (a) How could one conduct (out-of laboratory?) tests of the law-of-small-numbers?

   (b) How could one test the importance of crowding out of intrinsic motivation for regulatory incentive schemes?

   (c) Are limits to information processing an argument in favor or against auction mechanisms (in procurement and sales environments)?

(d) Is there a ”preference for truth-telling”?

5. Asymmetric Information in Industrial Organization

   (a) Can information asymmetries explain predatory pricing?
   (b) Can information asymmetries explain the price differential between first and second class railway tickets or business and economy class for regional flights?
   (c) What are the lessons to learn from the 3G telecom license allocation for trade institutions for airport slots or emission certificates?

D Further Literature

In the following, we list papers that might be a useful starting point for the final research project.

D.1 Empirical Contract Theory

1. *How to test for asymmetric information*. Chiappori and Salanie (2000) offer econometric techniques that detect asymmetric information in insurance contract relations. They apply the concepts to the French market for car liability insurance. In particular they discuss methodological shortcomings of preceding contributions.

2. *How to guarantee know-how transfer in a contractual relationship?* This is modelled and tested in Macho-Stadler et al. (1996). Their contribution offers a simple Moral Hazard model that explains certain structures in contracts for technology and know-how transfer. The authors test their theoretical predictions with data about relationships between Spanish and foreign firms.

3. *How to separate Moral Hazard and Adverse Selection*. Feess et al. (2004) develop a simple model for moral hazard and adverse selection to explain contract structures in professional sports leagues. They use legal changes in European soccer to disentangle Moral Hazard and Adverse Selection (Heterogeneity) effects and test their predictions with a dataset about the German "Bundesliga".

D.2 Regulations and Incentives

1. *How to regulate a natural monopoly.* In a landmark paper Laffont and Tirole (1986) establish the modern theory of incentive regulation. They apply standard screening techniques to characterize the optimal incentive scheme for a natural monopolist. In particular they explain when the optimal scheme is more similar to cost-plus or fixed-price regulation.

2. *How does regulation work in a political system?* In quite some publications such as Laffont (1996) the so-called Toulouse school of Political Economy (in contrast to the Chicago or Virginia school put forward e.g. by Becker (1985)) discussed regulatory performance in different political systems. The paper describes how public or private ownership of natural monopolies and the preferences of the (electoral) majority influences welfare.

3. *Is Incentive Regulation the Holy Grail?* Recently, papers like Falk and Koesfeld (2005) provided experimental evidence that high-powered incentive schemes may well crowd out (welfare enhancing) intrinsic motivations.

4. *Bypass and Cream Skimming in Regulated Telecommunication.* Laffont and Tirole (1990) investigate the impact of asymmetric information in the regulation of (partly liberalized) telecommunication markets.

D.3 (Strategic) Information Transmission and Processing

1. *What are the limits of information processing?* The seminal paper by Miller (1956) is a compilation of experimental evidence that suggests general limits to our cognitive capacities. A talk should not only review this important contribution but should also briefly mention recent confirmations and qualifications.

2. *Do we believe in the Law of Small Numbers?* Rabin (2000) provides a detailed review of experimental evidence starting with Kahneman and Tversky (1970) that suggest regular patterns of biased information updating. This paper also provides a model framework that allows for a structuring of the empirical findings and for an organized discussion of fields of application such as the market for financial analysts.

3. *The strategic value of advertisement*. Milgrom and Roberts (1986) apply signalling games to explain the advertisement and pricing patterns in markets for experience goods. This paper provides a theory of advertisement without transmission of product related information.

4. *What about just telling the truth?*. Social preferences for truth-telling are investigated in Sanchez-Pages and Vorsatz (2005). They elicit social preferences in a simple sender-receiver game, where receivers who detect cheating have the opportunity to punish.

D.4 Market Design and Information Aggregation

1. *What are the lessons to learn from the European auction of 3G telecom licenses?*. Many papers such as Klemperer (2002) apply key insights from auction theory to explain the heterogenous outcome of UMTS license auctions all over Europe and derive design suggestions for future rounds.

2. *How to detect collusion in auctions?* Stimulated by Porter and Zona (1993) and the subsequent economic literature, antitrust authorities start to use econometric techniques that provide additional evidence for bid rigging in procurement auctions.

3. *What can economists learn from eBay and vice versa - Part 1.* Reily (2005) tests some fundamental predictions of auction theory in a field experiment on eBay.

4. *What can economists learn from eBay and vice versa - Part 2*. Roth and Ockenfels (2002) (and Ockenfels and Roth (2005)) investigate theoretically and empirically the phenomenon of late bidding on the internet.

E Refresher in Expected Utility Theory

This section offers a precise formulation of expected utility theory. Every participant of the course should make himself familiar with the concept of a utility function (i.e. its relation to individual preferences) and should be aware of the conditions when preferences over lotteries can be represented in expected utility form.

E.1 Lotteries

Let C be the set of so–called elementary outcomes. In principle, an elementary outcome may have any nature. In this course, however, we will be mainly concerned with outcomes having the form of monetary payoffs. You can therefore think of elementary outcome c as specifying an amount of money and identify the set C with the set of all (real) numbers. Sometimes we will allow for positive payoffs only, in which case C is the set of positive numbers.

An important notion in the theory of decision making under uncertainty is that of a *lottery*, also called a *prospect* or a *gamble*. A simple lottery p specifies finitely many of uncertain outcomes $c_1, c_2, \ldots, c_K$ together with the probabilities $p(c_1), p(c_2), \ldots, p(c_K)$ of each outcome. The probabilities $p(c_k)$ are non–negative numbers adding up to 1. We denote the set of all simple lotteries by $\mathcal{P}$. A simple lottery p can be thought of as a probability distribution on the set of elementary outcomes C which assigns a positive probability to at most a finite number of outcomes.

Let p be a simple lottery which assigns probability $\frac{1}{2}$ to each of the outcomes 0 and 1. Let q be another simple lottery where outcomes -1 and 0 occur with probabilities $\frac{1}{4}$ and $\frac{3}{4}$. Let $(p, q; \alpha, 1-\alpha)$ denote a compound (or complex) lottery in which one receives simple lottery p with probability α and q otherwise. Figure 2 gives a schematic representation of a compound lottery. It should be clear that compound lottery $(p, q; \alpha, 1-\alpha)$ is equivalent to the simple lottery r with outcomes 0, -1, and 1 and probabilities

$$\begin{aligned} r(0) &= \alpha \times 1/2 &+& (1-\alpha) \times 3/4 \\ r(-1) &= \alpha \times 0 &+& (1-\alpha) \times 1/4 \\ r(1) &= \alpha \times 1/2 &+& (1-\alpha) \times 0. \end{aligned}$$

More generally, if p and q are simple lotteries then the compound lottery $(p, q; \alpha, 1-\alpha)$ is equivalent to the simple lottery r which assigns probability $r(c) = \alpha p(c) + (1-\alpha) q(c)$ to every elementary outcome c in the set C.

E.2 Preferences and utility functions

The tastes of the individual with regard to lotteries are summarized by the preference relation $\succeq$ on $\mathcal{P}$. We assume that for any pair p, q of simple lotteries an individual is able to decide whether p is preferred to q (in which case we write $p \succ q$), q is preferred to p ($q \succ p$), or he/she is indifferent between p and q ($p \sim q$). Compound lotteries can be compared on the basis of equivalent simple lotteries.

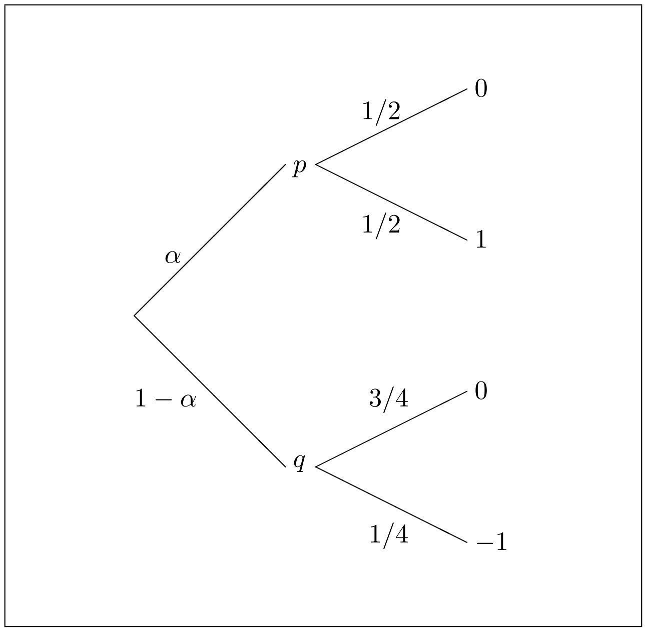

Figure 1: Compound lottery $(p, q; \alpha, 1-\alpha)$

The utility function is another way to describe the tastes of an individual. The utility function U is said to *represent* preference relation $\succeq$ over $\mathcal{P}$ if for any pair of simple lotteries p and q it holds that

$$\text{if } p \succ q \text{ then } U(p) > U(q)$$
$$\text{if } p \sim q \text{ then } U(p) = U(q)$$

One of the central results in microeconomics states that any preference relation satisfying a number of technical and rather unrestrictive assumptions admits a representation by the utility function. Typically, such a representation is not unique. For suppose U is the utility representation of the preference relation $\succeq$. Choose any strictly increasing function f such as logarithmic ($f(x) = \ln(x)$), exponential ($f(x) = e^x$), or linear ($f(x) = ax + b$ where a is a positive number and b is any number). Define function W on the set $\mathcal{P}$ by setting $W(p)$ equal to $f(U(p))$. Function W is then said to be obtained by a monotone transformation of U. It is easy to see that W represents the same preference relation $\succeq$ over the set $\mathcal{P}$. Indeed, if $p \succ q$ then $U(p) > U(q)$ since U is the utility representation of $\succeq$. As f is a strictly increasing function, we have $f(U(p)) > f(U(q))$. So, W provides a correct ordering of the alternatives p and q. This observation is summarized by the following lemma.

Lemma 1 *Let U be utility function representing the preference relation $\succeq$ on $\mathcal{P}$. Then any function obtained by a monotone transformation of U represents the same preference relation $\succeq$.*

E.3 The expected utility theorem

Definition 1 The preference relation $\succeq$ satisfies the axiom of complex gambles if for any three simple lotteries p, q, r, and any number $\alpha \in (0,1)$ it holds that

$$\text{if } p \succ q \text{ then } (p, r; \alpha, 1-\alpha) \succ (q, r; \alpha, 1-\alpha)$$
$$\text{if } p \sim q \text{ then } (p, r; \alpha, 1-\alpha) \sim (q, r; \alpha, 1-\alpha)$$

Definition 2 The utility function U on $\mathcal{P}$ has the expected utility form if there exists a function v on C such that the utility of any simple lottery $(c_1, \ldots, c_K, p)$ is

$$U(p) = \sum_{k=1}^{K} p(c_k) v(c_k). \tag{1}$$

Equation (2) essentially says that the utility of a lottery p is computed as the mathematical expectation of the values $v(c_1), \ldots, v(c_K)$. Function v in the above definition is called the *preference–scaling function* or an *elementary utility function*. Let us emphasis a crucial distinction between U and v: **Function U is defined on the set $\mathcal{P}$ of simple lotteries. Function v is defined on the set C of elementary outcomes**.

We proceed to study the relationship between Definitions 3 and 2. In fact, these are equivalent in the following sense: a preference relation $\succeq$ satisfies the axiom of complex gambles if and only if it admits a representation by the utility function of the expected utility form. For convenience, we split this proposition in two parts.

Theorem 1 *Let $\succeq$ be a preference relation on $\mathcal{P}$ represented by the utility function U. Suppose that U has the expected utility form. Then $\succeq$ satisfies the axiom of complex gambles.*

To prove Theorem 3 choose three simple lotteries p, q, and r. If we let t be a simple lottery equivalent to $(p, r; \alpha, 1-\alpha)$ then

$$\begin{aligned} U(t) &= \sum t(c)v(c) \\ &= \sum [\alpha p(c) + (1-\alpha) r(c)] v(c) \\ &= \alpha \sum p(c)v(c) + (1-\alpha) \sum r(c)v(c) \\ &= \alpha U(p) + (1-\alpha) U(r), \end{aligned}$$

where the sums are taken over all those outcomes c that either $p(c)$ or $q(c)$ is positive. Similarly, if w is a simple lottery equivalent to $(q, r; \alpha, 1-\alpha)$ then

$$U(w) = \alpha U(q) + (1-\alpha) U(r).$$

Suppose that $p \succ q$. Then $U(p) > U(q)$, which clearly implies the inequality $U(t) > U(w)$. The compound lottery $(p, r; \alpha, 1-\alpha)$ is therefore preferred to $(q, r; \alpha, 1-\alpha)$, as required by Definition 3. The case $p \sim q$ is dealt with similarly.

Theorem 2 (The expected utility theorem) *Let $\succeq$ be a preference relation on $\mathcal{P}$ satisfying the axiom of complex gambles. Then $\succeq$ admits a representation by the utility function of the expected utility form.*

We do not present the proof Theorem 2. The idea is to construct function v using the reference–lottery technique explained in Section 1.4.1.

Example 1 Suppose that the preference relation $\succeq$ is represented by the utility function

$$U(p) = \ln\left[\sum_{k=1}^{K} p(c_k)c_k\right].$$

Does $\succeq$ satisfy the axiom of complex gambles? Define function W on the set $\mathcal{P}$ by $W(p) = e^{U(p)}$. By Lemma 1, function W is the utility representation of $\succeq$. Moreover, W has the expected utility form, with the function v given by $v(c) = c$. Theorem 3 now implies that $\succeq$ does satisfy the axiom of complex gambles.

E.4 Application: the utility function over actions

In this section we demonstrate that the preference relation over the set of simple lotteries gives rise to the utility function over actions in a natural way.

As in Sections 1.1–1.3 of Hirshleifer and Riley let $1, \ldots, S$ be the states of the world, X be a set of actions available to an individual, and suppose that the consequence function $c : (x, s) \mapsto c(x, s)$ is specified. "To choose an act is to choose one of the rows of the consequence matrix like table 1.1 on page 8. Each such row can be regarded as a probability distribution", or a lottery. Thus a lottery associated with action x is $(c_{x1}, \ldots, c_{xS}; \pi_1, \ldots, \pi_S)$.

Now, suppose that the preferences of an individual over the set $\mathcal{P}$ are represented by the utility function U. It is natural to define the utility of action x as that of the associated lottery $(c_{x1}, \ldots, c_{xS}; \pi_1, \ldots, \pi_S)$. In this way, the preference relation on $\mathcal{P}$ induces the ordering of actions.

In a special case where $\succeq$ satisfies the axiom of complex gambles we can take U to be the function of the expected utility form. The utility of action x can then be computed as

$$U(x) = \sum_{s=1}^{S} \pi_s v(c_{xs}).$$

F Refresher in Risk Aversion

We assume throughout this note that the preference relation of an individual over the set of simple lotteries admit a representation by the expected utility function U. The utility of the lottery p with outcomes $c_1, \ldots, c_K$ is therefore given by

$$U(p) = \sum_{k=1}^{K} p(c_k) v(c_k). \tag{2}$$

It is often convenient to rewrite equation (2) as

$$U(p) = E_p(v(c)), \tag{3}$$

where the symbol E_p denotes the mathematical expectation under probability distribution p.

Lottery p is said to be non–trivial (or risky) if at least two distinct elementary outcomes occur under p with positive probabilities. That if, a non–trivial lottery involves some uncertainty. We shall say that lottery q is trivial if $q(\bar{c}) = 1$ for some elementary outcome $\bar{c}$ and $q(c) = 0$ for any $c \neq \bar{c}$.

Definition 3 The preferences of the individual are said to exhibit risk–aversion if for every non–trivial simple lottery p the following inequality holds:

$$E_p(v(c)) < v(E_p(c)). \tag{4}$$

To understand the above definition it is crucial to clearly see the difference between the following expressions:

(a) $E_p(v(c))$. This was defined above as a mathematical expectation of the random variable which assumes the values $v(c_1), \ldots, v(c_k)$.

(b) $E_p(c)$. This is the mathematical expectation of the random variable c under probability distribution p:

$$E_p(c) = \sum_{k=1}^{K} p(c_k) c_k.$$

(c) $v(E_p(c))$. This is the value of function v at the point $E_p(c)$.

The left–hand side of equation (4) gives the expected utility of lottery p. The right–hand side of (4) is the utility of consuming $E_p(c)$ units of income with probability one. Thus, according to Definition 3, an individual is risk–averse if participating in a risky lottery is considered less attractive than receiving the expected income of this lottery with certainty.

Example 2 Let p be a lottery which assigns probability $\frac{1}{2}$ to each of the outcomes 0 and 4. Suppose that $v(c) = \sqrt{c}$. Then

$$\begin{aligned} E_p(v(c)) &= \tfrac{1}{2} \times v(0) + \tfrac{1}{2} \times v(4) = 1 \\ E_p(c) &= \tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 4 = 2 \\ v(E_p(c)) &= v(2) = \sqrt{2}. \end{aligned}$$

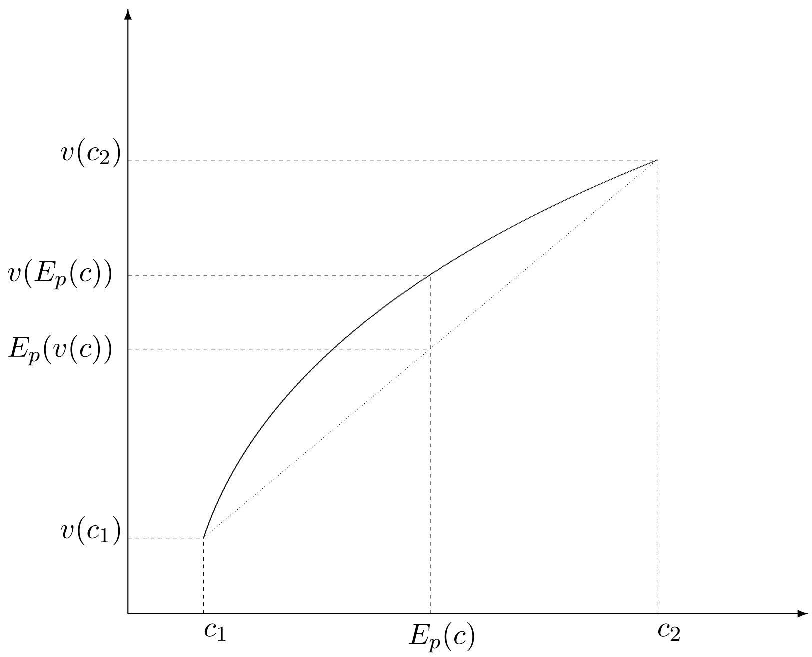

Figure 2: The graph of function v

Since $\sqrt{2} > 1$, we conclude that the individual in question prefers to receive two units of income with probability 1 than to participate in the lottery where he/she receives 0 or 4 units of income with equal probabilities. Thus, inequality (4) is satisfied, and the individual is indeed risk-averse (in fact, one should verify that inequality (4) holds for every lottery p).

We remark that function v satisfying the condition of Definition 3 is called *strictly concave*. In the theorem below $v''(c)$ denotes the second derivative of v at point c.

Theorem 3 *Suppose that function v is twice continuously differentiable. Then the agent's preferences exhibit risk aversion if and only if $v''(c) < 0$ for any elementary outcome c.*

We now turn to the graphical representation of the risk–aversion hypothesis.

In Figure 2 the horizontal axis measures the elementary outcomes (i.e. payoffs) c, while the vertical axis measures the values of function v. A bold curve is a graph of function v. Two possible outcomes of lottery p are indicates as c_1 and c_2. We assume that both of these occur with probability $\frac{1}{2}$, so that the expected payoff $E_p(c)$ of lottery p is located exactly at the midpoint of the interval $[c_1, c_2]$. Correspondingly, the expected utility of p, $E_p(v(c))$, is a midpoint of the interval $[v(c_1), v(c_2)]$. Since the expected utility of p is smaller than that of a certain income $E_p(c)$, we conclude that the individual characterized by function v as depicted in Figure 2 is a risk–averse agent.

G Refresher in Constrained Optimization

A consumer consumes two commodities, wheat and candy. If w is the amount of wheat consumed by this individual and c is the amount of candy, the consumer's utility will be given by $u(w, c) = 3\ln(w) + 2\ln(c+2)$. The individual maximizes his utility subject to the following constraints: (i) the amount of candy and wheat have to be nonnegative, (ii) the individual has ten euros to spend and prices are 1 euro per unit of candy or wheat, (iii) a unit of wheat contains 150 calories and a unit of candy 200 calories, and the consumer is constrained to eat no more than 1550 calories.

1. Formulate the optimization problem.
2. Form the Lagrangian.
3. Elicit the first-order conditions.
4. Derive the solution.
5. Consider the more general problem of a firm with production technology $u(w, c)$ (i.e., the firm uses inputs w and c and produces an amount of $u(w, c)$) and the following budget constraint: The firm has a budget B and prices for w and c are p_w and p_c, respectively. Derive (along the lines of the previous analysis) an equation which fixes the optimal combination of wheat and candy for a given set of prices p_w and p_c. provide an economic interpretation.